Superconductivity - Overview

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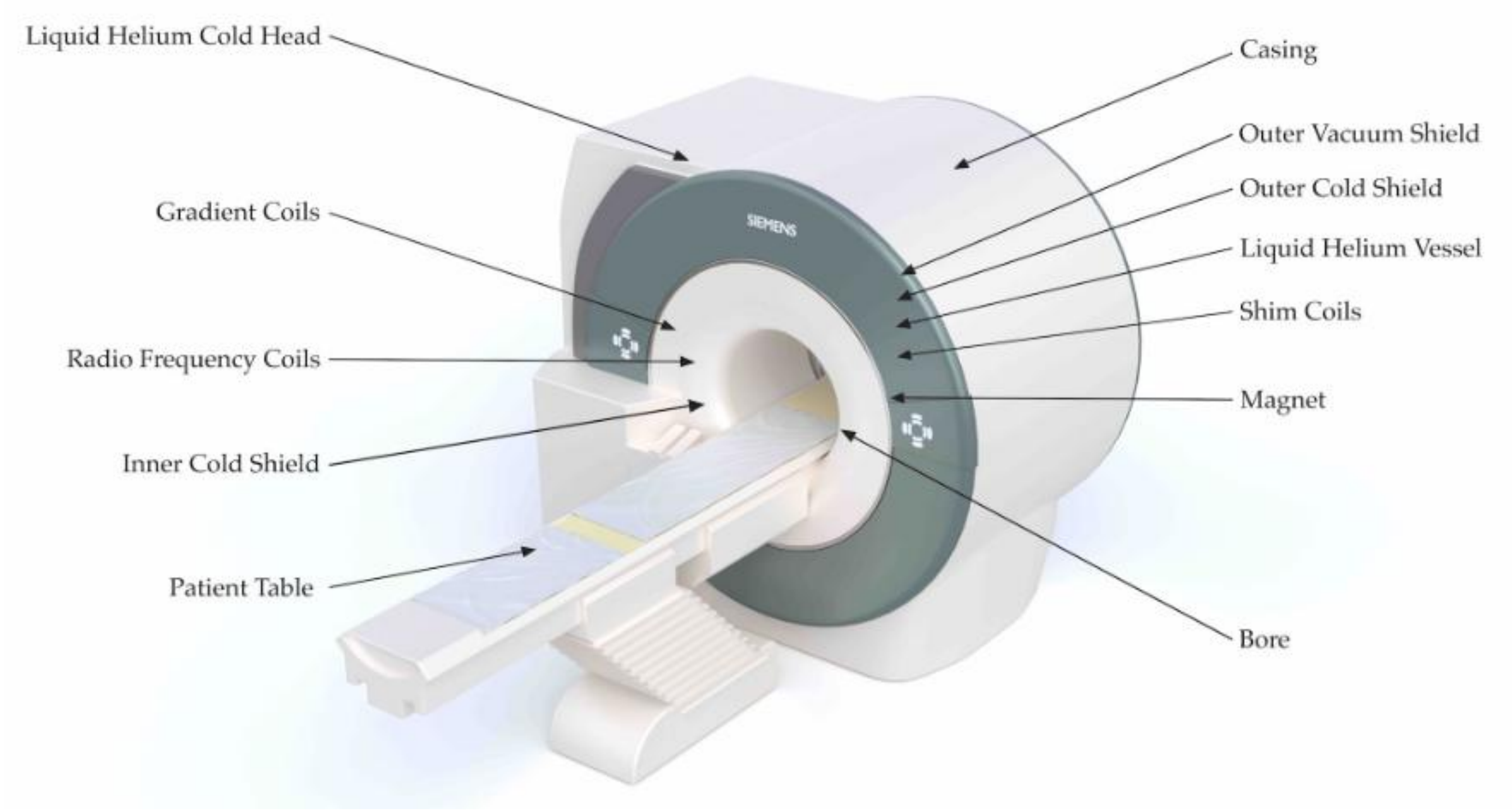
Magnetic Resonance Imaging (MRI)



https://socal-engineer.com/engineering-blog/2023/6/23/mri-technologies-overview

[MRI Technologies Overview — So-Cal Engineer](https://socal-engineer.com/engineering-blog/2023/6/23/mri-technologies-overview)

Superconducting magnets for MRI

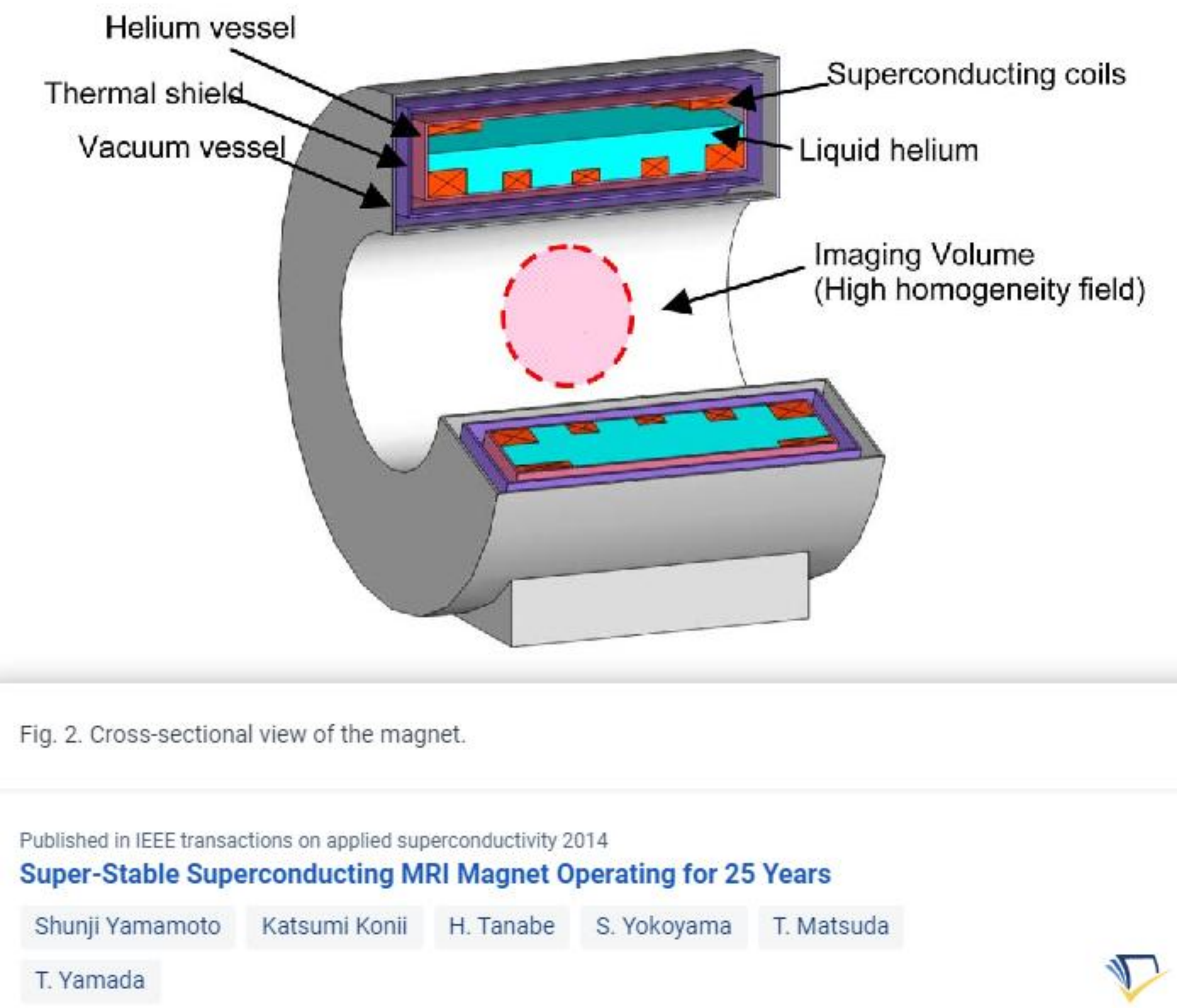


Fig. 2. Cross-sectional view of the magnet.

Published in IEEE transactions on applied superconductivity 2014

Super-Stable Superconducting MRI Magnet Operating for 25 Years

Shunji Yamamoto | Katsumi Konii | H. Tanabe | S. Yokoyama | T. Matsuda | T. Yamada

Superconducting-magnet levitation train

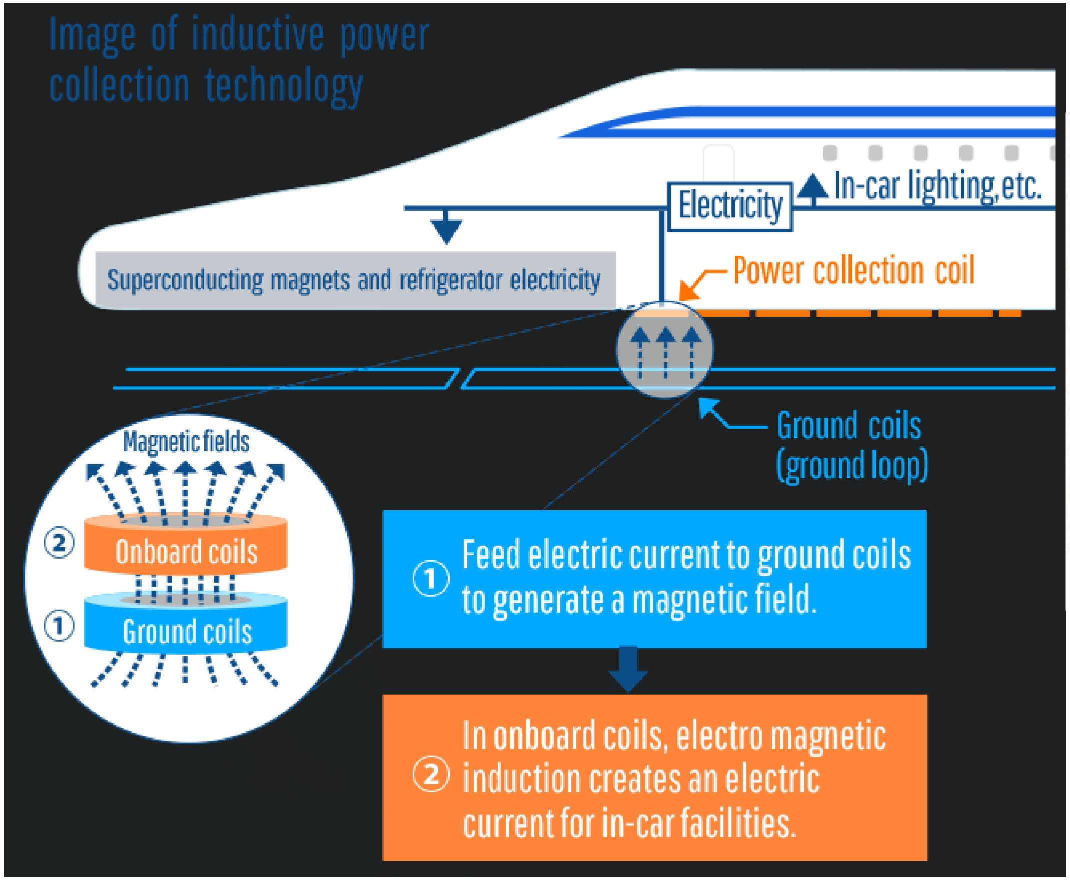


Image of inductive power collection technology

① Feed electric current to ground coils to generate a magnetic field.

② In onboard coils, electro magnetic induction creates an electric current for in-car facilities.

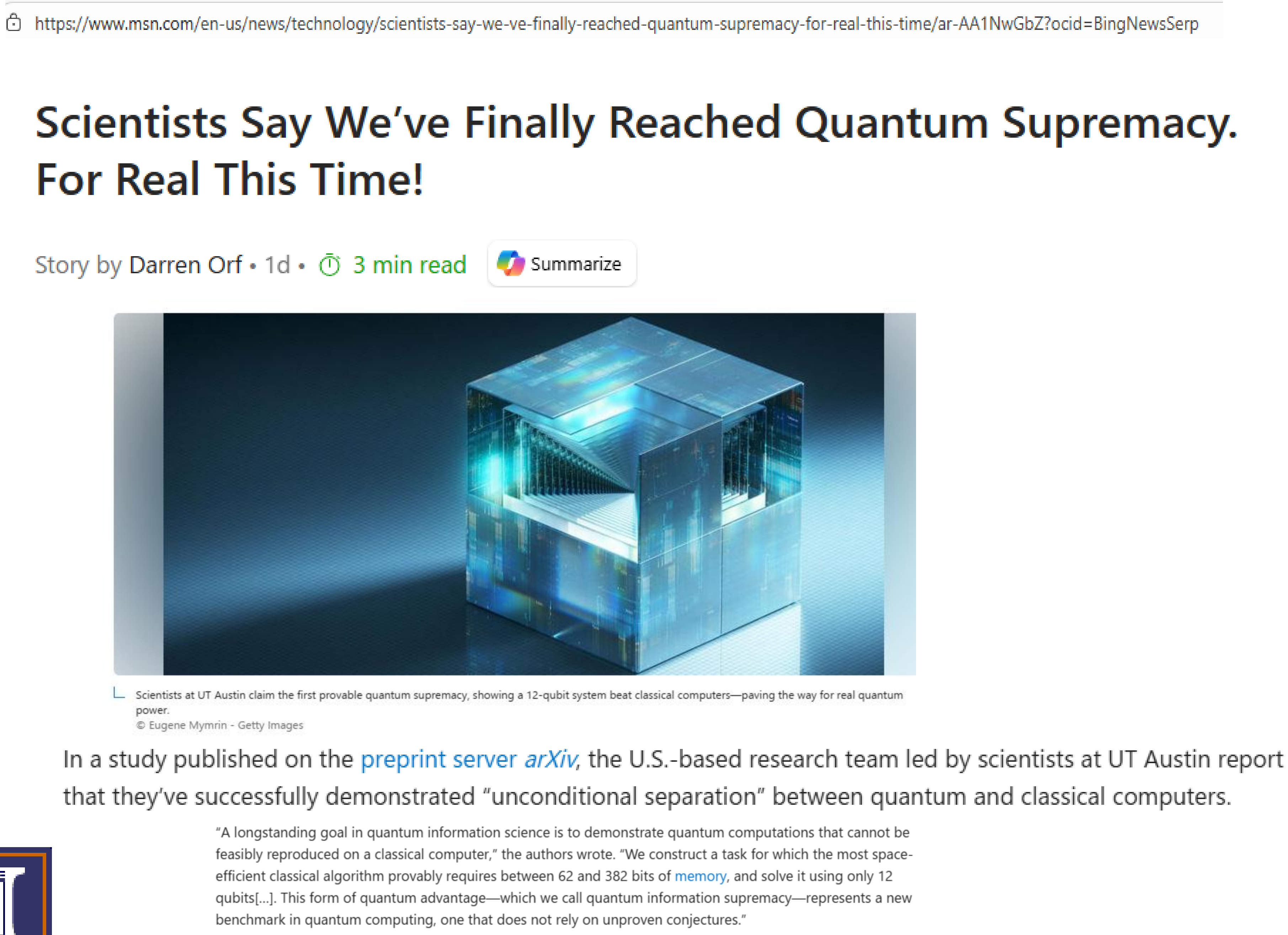
Japan's Maglev Train

The L0 Series, a prototype vehicle based on SCMaglev technology, holds the record for fastest crewed rail vehicle with a record speed of 603 km/h (375 mph).
Time Urbana-Chicago: only ~25 min

The SCMaglev system uses an electrodynamic suspension (EDS) system. The train's bogies have superconducting magnets installed, and the guideways contain two sets of metal coils.

Importance of superconductivity: Qubits and modern quantum computers are made of superconductors

IBM Q



https://www.msn.com/en-us/news/technology/scientists-say-we-ve-finally-reached-quantum-supremacy-for-real-this-time/ar-AA1NwGbZ?ocid=BingNewsSerp

Scientists Say We've Finally Reached Quantum Supremacy. For Real This Time!

Story by Darren Orf • 1d • 3 min read

Scientists at UT Austin claim the first provable quantum supremacy, showing a 12-qubit system beat classical computers—paving the way for real quantum power.
© Eugene Mymrin - Getty Images

In a study published on the preprint server *arXiv*, the U.S.-based research team led by scientists at UT Austin report that they've successfully demonstrated "unconditional separation" between quantum and classical computers.

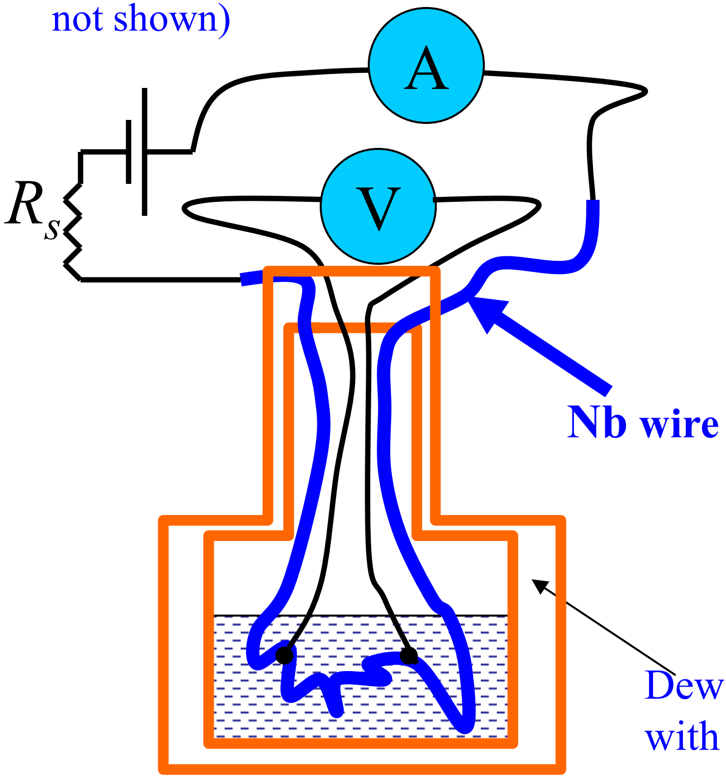
> "A longstanding goal in quantum information science is to demonstrate quantum computations that cannot be feasibly reproduced on a classical computer," the authors wrote. "We construct a task for which the most space-efficient classical algorithm provably requires between 62 and 382 bits of memory, and solve it using only 12 qubits[...]. This form of quantum advantage—which we call quantum information supremacy—represents a new benchmark in quantum computing, one that does not rely on unproven conjectures."

How to measure superconducting transitions

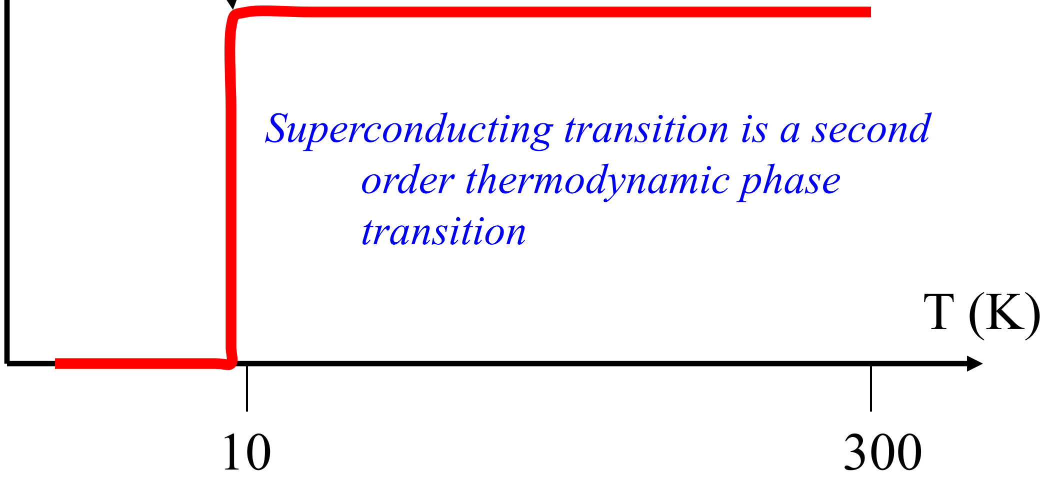
Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature“ (H. K. O. 1911)

How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T) (thermometer is not shown)

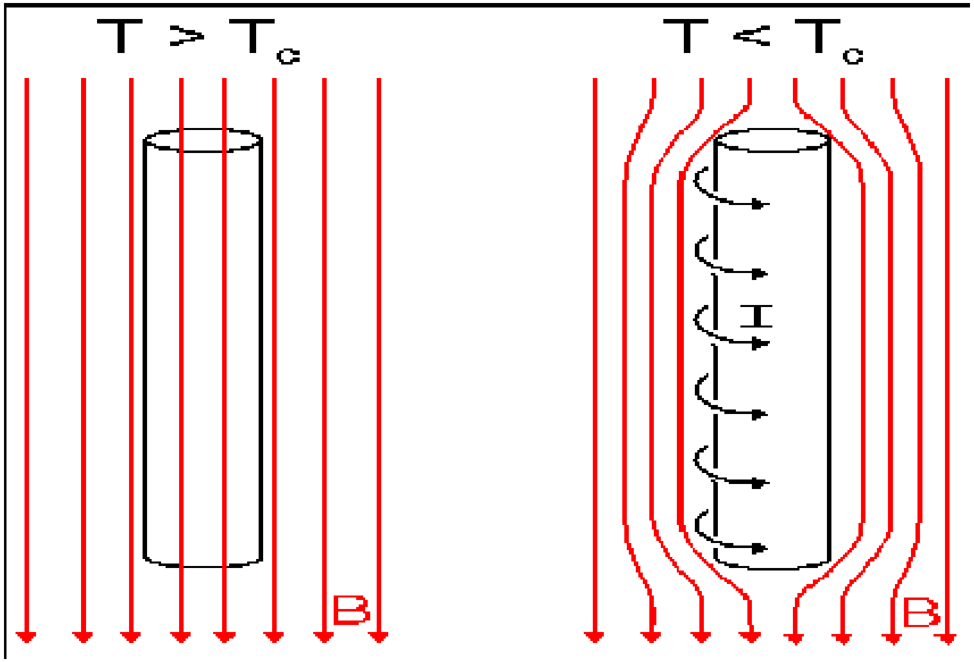


Heike Kamerling Onnes

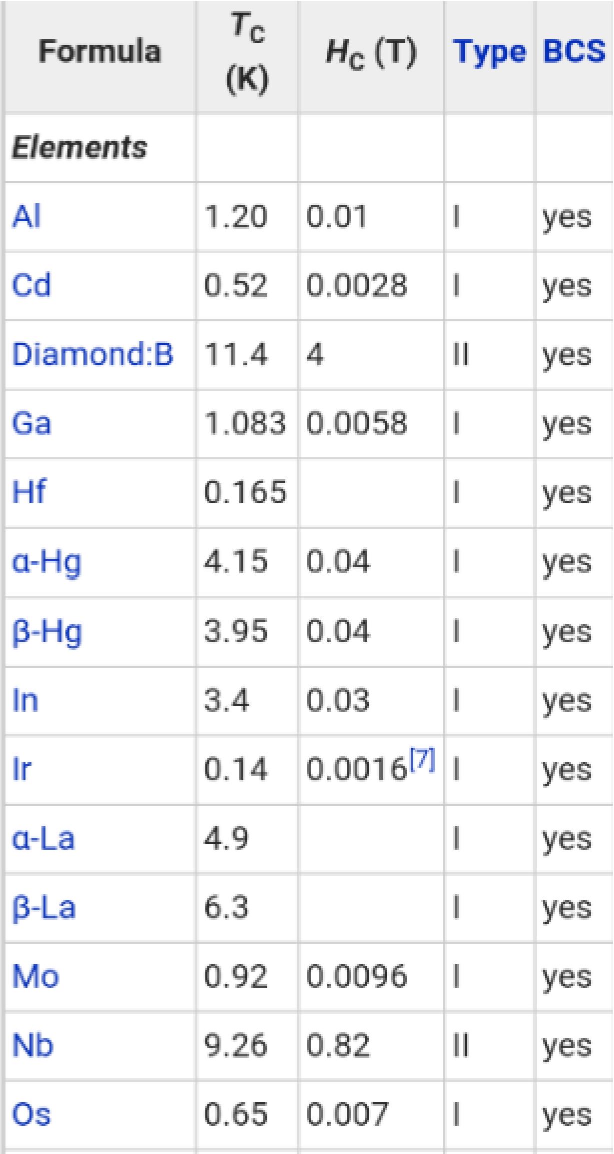


Superconducting transition is a second order thermodynamic phase transition

Meissner effect – the key signature of superconductivity

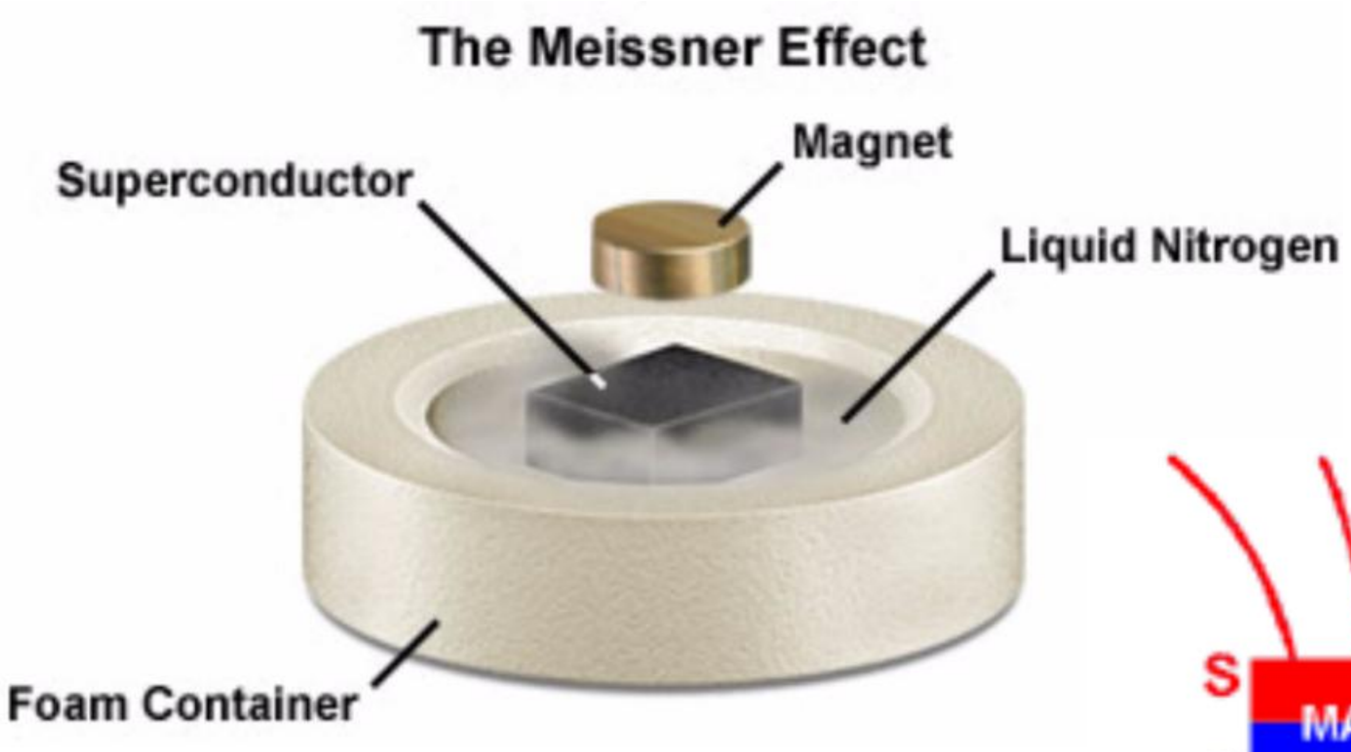


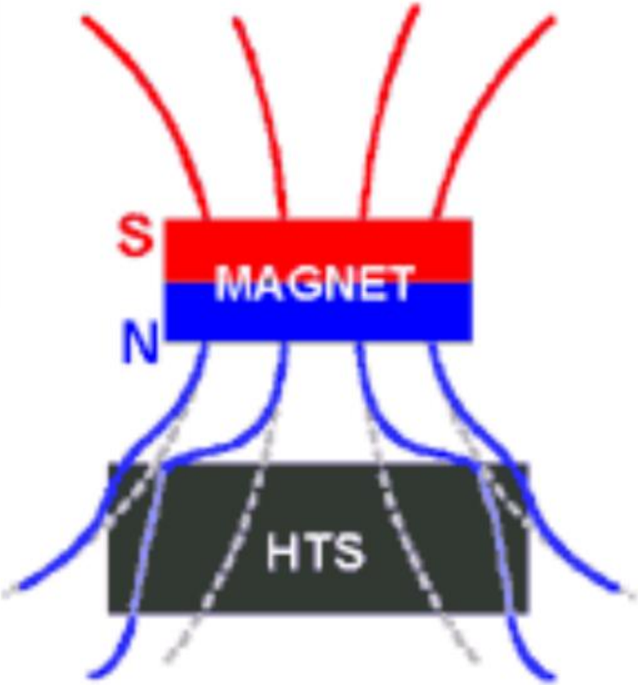
Theory of superconductivity:
“BCS” – due to Bardeen, Cooper and Schrieffer



Formula	T_c (K)	H_c (T)	Type	BCS
Elements				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α-Hg	4.15	0.04	I	yes
β-Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016[7]	I	yes
α-La	4.9		I	yes
β-La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

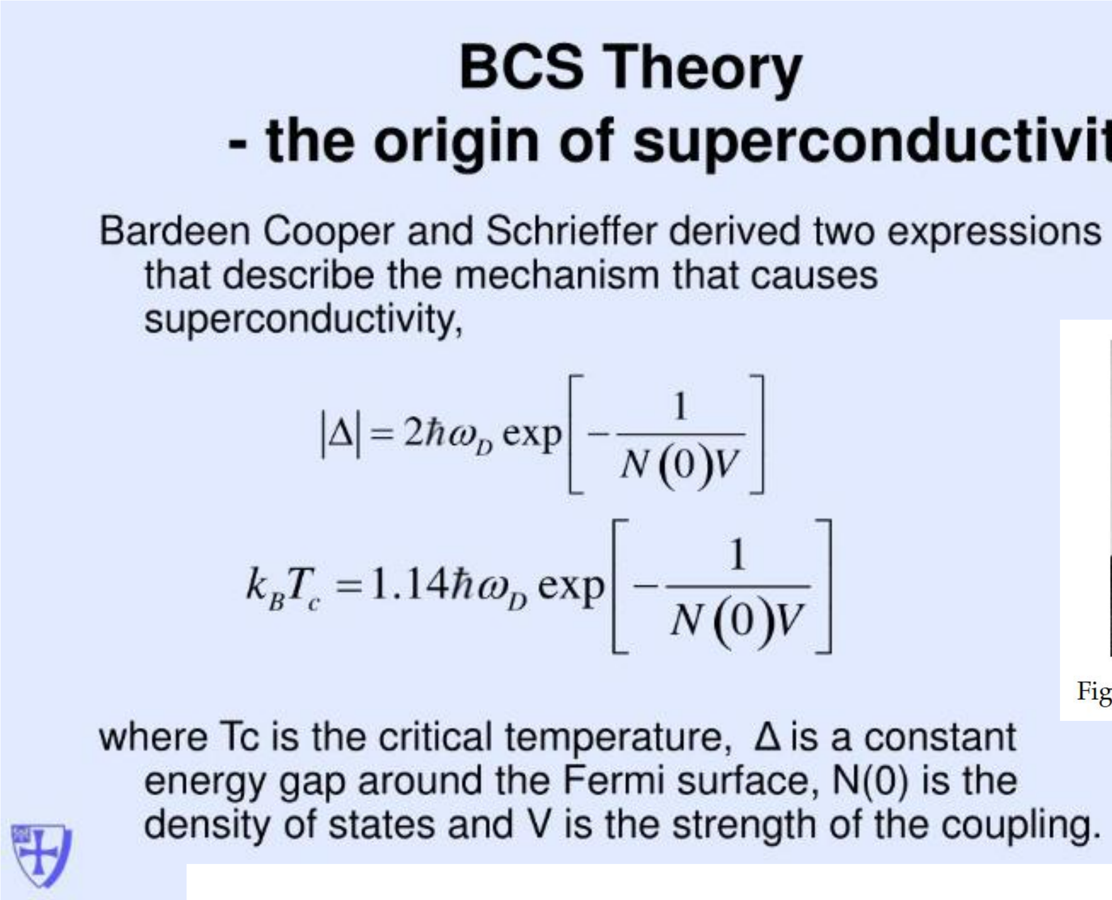
Interesting phenomenon: Magnetic levitation





Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.

https://nationalmaglab.org/magnet-academy/read-science-stories/science-simplified/superconductivity-101/



BCS Theory
- the origin of superconductivity

Bardeen Cooper and Schrieffer derived two expressions that describe the mechanism that causes superconductivity,

$$|\Delta| = 2\hbar\omega_D \exp\left[-\frac{1}{N(0)V}\right]$$

$$k_B T_c = 1.14\hbar\omega_D \exp\left[-\frac{1}{N(0)V}\right]$$

where Tc is the critical temperature, Δ is a constant energy gap around the Fermi surface, N(0) is the density of states and V is the strength of the coupling.

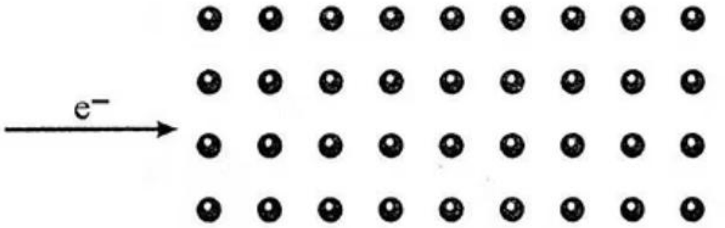
Figure 12.3: John Bardeen, Leon Cooper, and J. Robert Schrieffer.

Near Tc: $\Delta(T) = 1.734\,\Delta(0)\left(1 - \frac{T}{T_\mathrm{c}}\right)^{1/2} \simeq 3.06\,k_\mathrm{B}T_\mathrm{c}\left(1 - \frac{T}{T_\mathrm{c}}\right)^{1/2}$

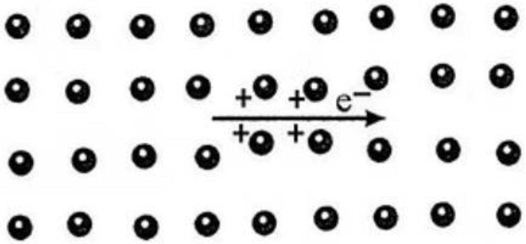
https://courses.physics.ucsd.edu/2023/Spring/physics239/LECTURES/C12.pdf

Attraction between the electrons is needed for superconductivity. It is provided by phonons.

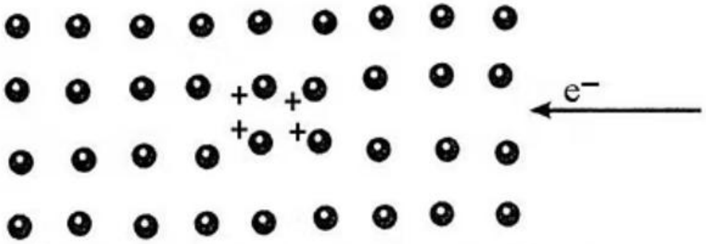
Debye Frequency ω_D



First electron approach a section of the crystal lattice



First electrons defoms part of the lattice electrostatically.



Second electron is attracted to the net positive charge of the deformation

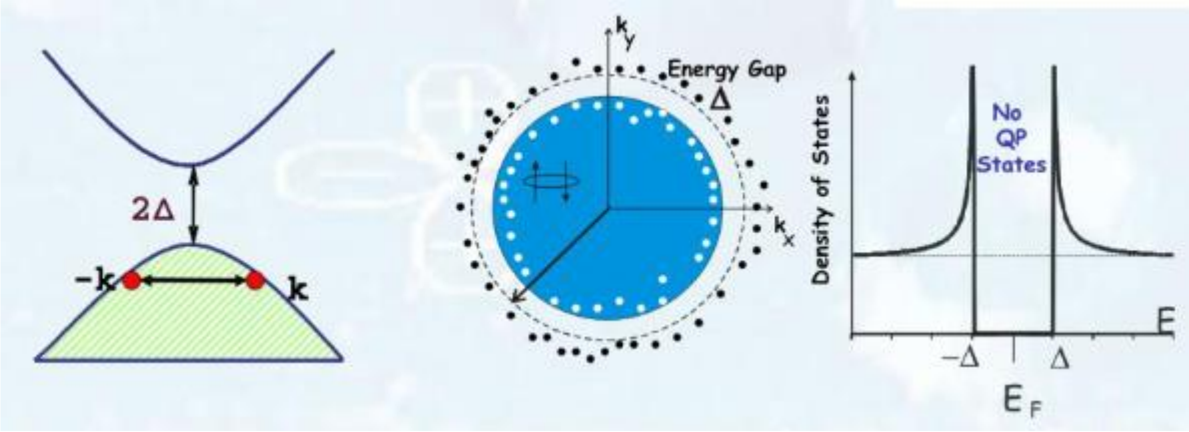
The Debye frequency represents the highest vibrational frequency of the atoms in the crystal lattice of the superconductor. There are no phonon modes above this frequency. Thus, Debye frequency is the upper limit of the phonon spectrum in the Debye model of specific heat.

https://www.physicspower.com/post/bcs-theory

BCS theory

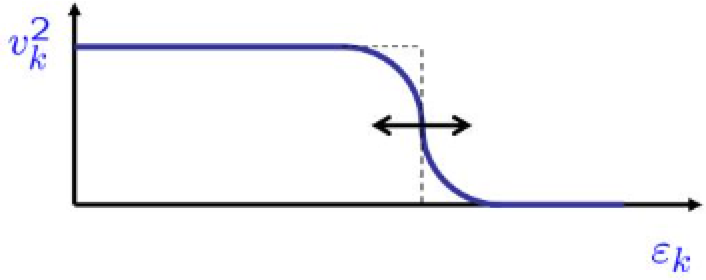
- Gap and order parameter

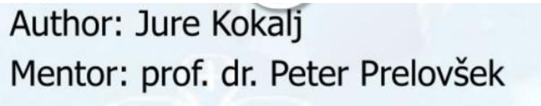
https://phys.uri.edu/~nigh/Leuven-2011/henley/7.3.pdf



$$\boxed{|BCS\rangle = \prod_k (u_k + v_k a^\dagger_k a^\dagger_{-k})|-\rangle}$$

$$u_k^2 + v_k^2 = 1$$





Author: Jure Kokalj
Mentor: prof. dr. Peter Prelovšek

https://www.slideserve.com/connie/unconventional-superconductivity

BCS theory

- Gap and order parameter

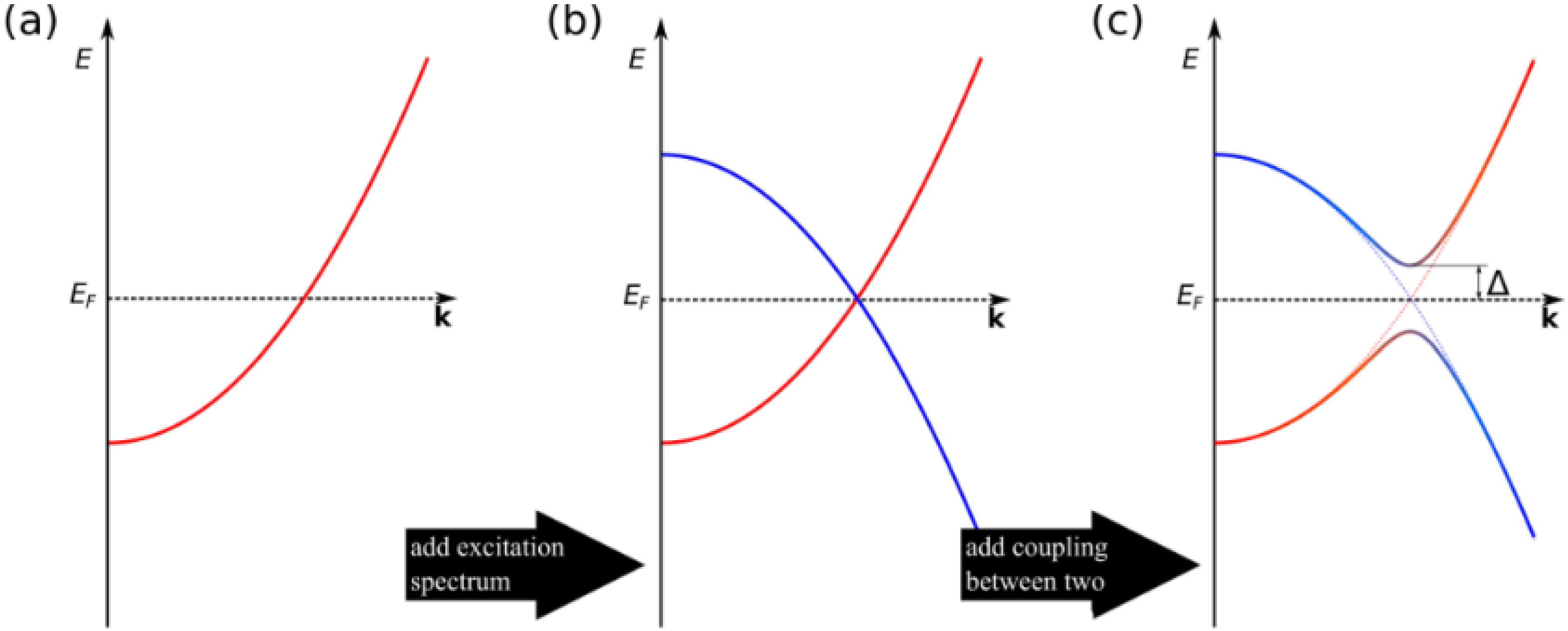


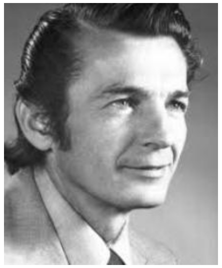
*Figure 1: **BCS theory: example of electron and hole bands coupling.** (a) a parabolic electron-like band, (b) add the excitation hole-like band, (c) superconducting gap opens when the electron-like and hole-like bands are coupled by an interaction Δ.*

https://shunchi100.wordpress.com/superconductivity-and-its-pairing-symmetry/mean-field-approximation-to-bcs-hamiltonian/

Energy gap in superconductors: Giaever junction experiment

Near Tc:

$$\Delta(T) = 1.734\,\Delta(0)\left(1-\frac{T}{T_{\rm c}}\right)^{1/2} \simeq 3.06\,k_{\rm B}T_{\rm c}\left(1-\frac{T}{T_{\rm c}}\right)^{1/2}$$



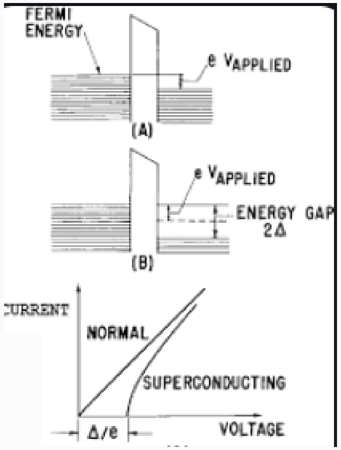
Artificial intelligence advice:

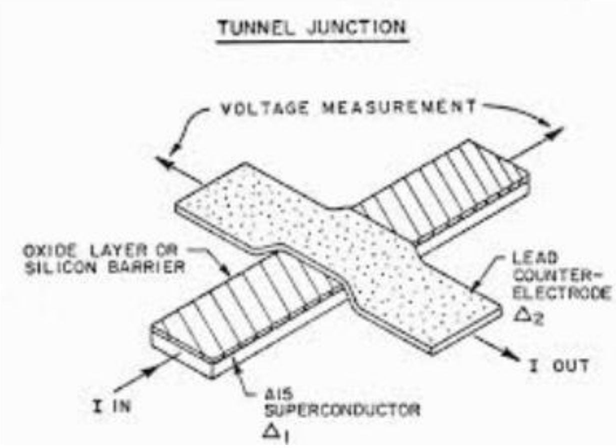
The temperature dependence of the energy gap ($\Delta(T)$) in superconductors over the entire temperature range can be described by a more precise formula derived from the BCS (Bardeen-Cooper-Schrieffer) theory. The formula is:

$$\Delta(T) = \Delta(0)\tanh\left(\frac{\pi k_B T_c}{\Delta(0)}\sqrt{\frac{T_c}{T}-1}\right)$$

where:

- $\Delta(0)$ is the energy gap at absolute zero temperature,
- T is the temperature,
- T_c is the critical temperature,
- k_B is the Boltzmann constant.





If the oxide barrier in the tunnel junction is sufficiently thin then one gets Josephson junction, which has zero resistance

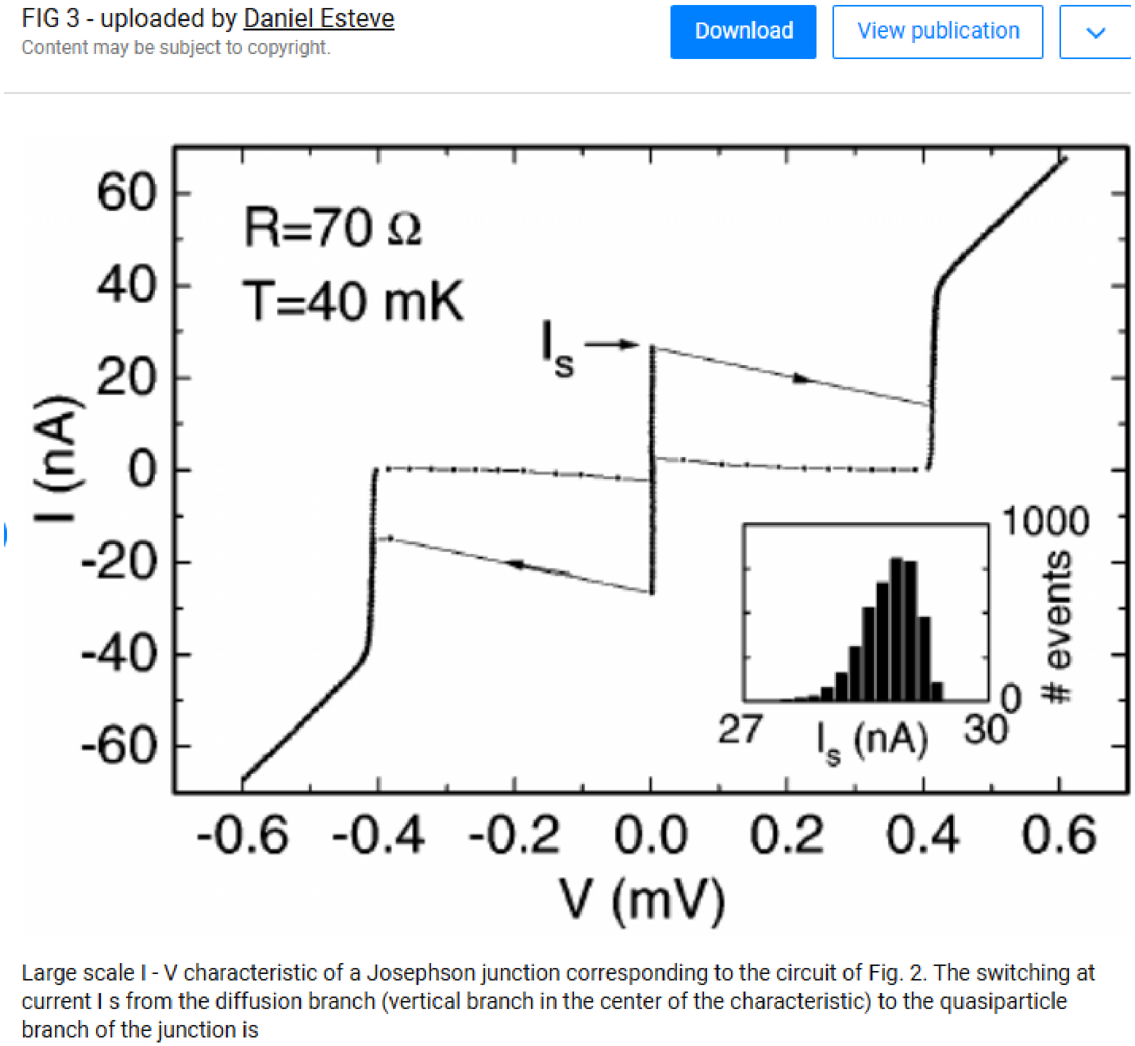
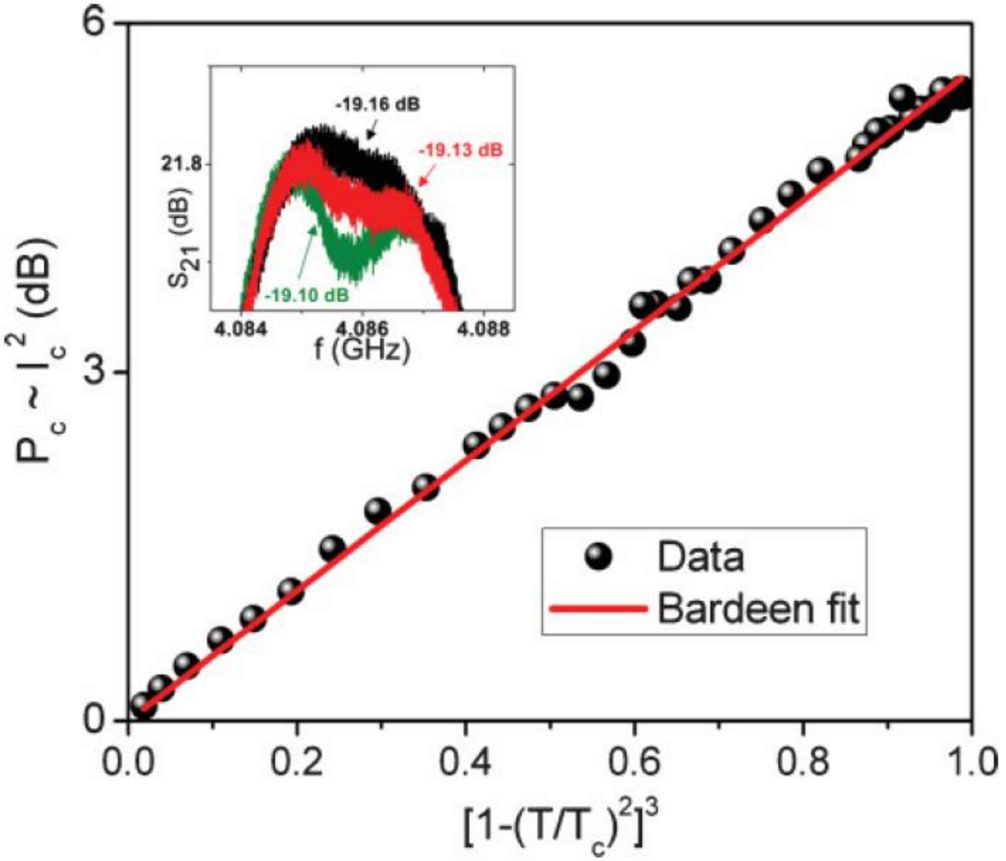


FIG 3 - uploaded by Daniel Esteve

Large scale I - V characteristic of a Josephson junction corresponding to the circuit of Fig. 2. The switching at current I s from the diffusion branch (vertical branch in the center of the characteristic) to the quasiparticle branch of the junction is

Critical current of a thin superconducting wire. Bardeen formula.

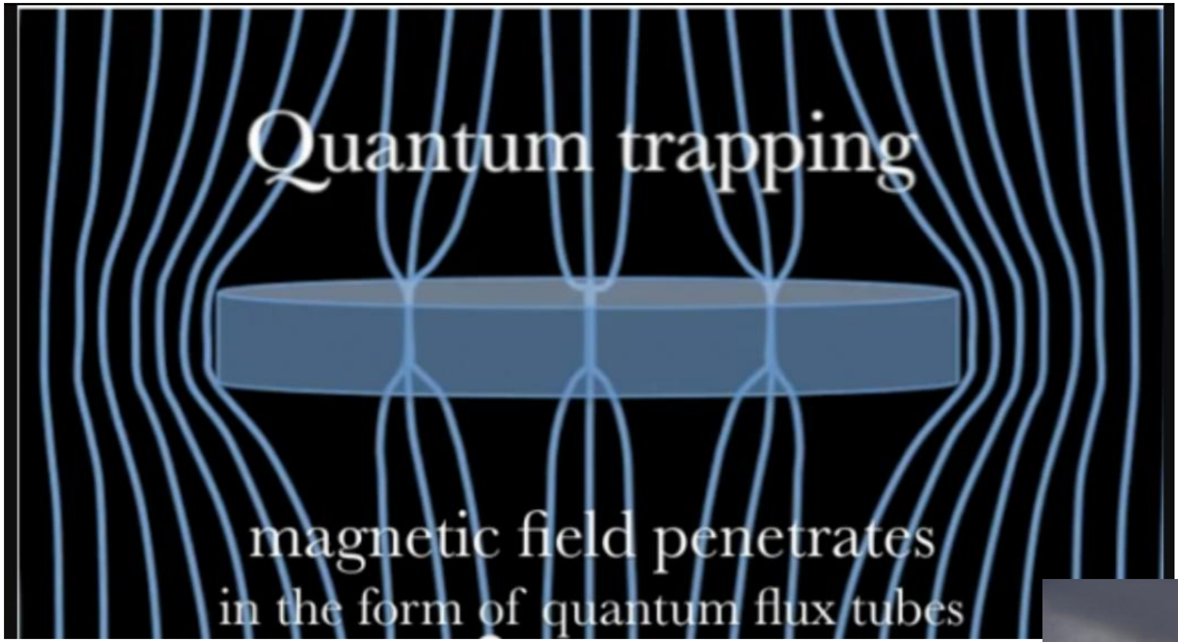


M. W. Brenner, S. Gopalakrishnan, J. Ku, T. J. McArdle,J. N. Eckstein,
N. Shah, P. M. Goldbart, and Alexey Bezryadin, “Cratered Lorentzian response of driven microwave superconducting nanowire-bridged resonators:
Oscillatory and magnetic-field induced stochastic states”,
PHYSICAL REVIEW B 83, 184503 (2011)

$$I_c = I_c(0)[1 - (T/T_c)^2]^{3/2}$$

https://research.physics.illinois.edu/bezryadin/publication/PhysRevB.83.184503_Brenner.pdf

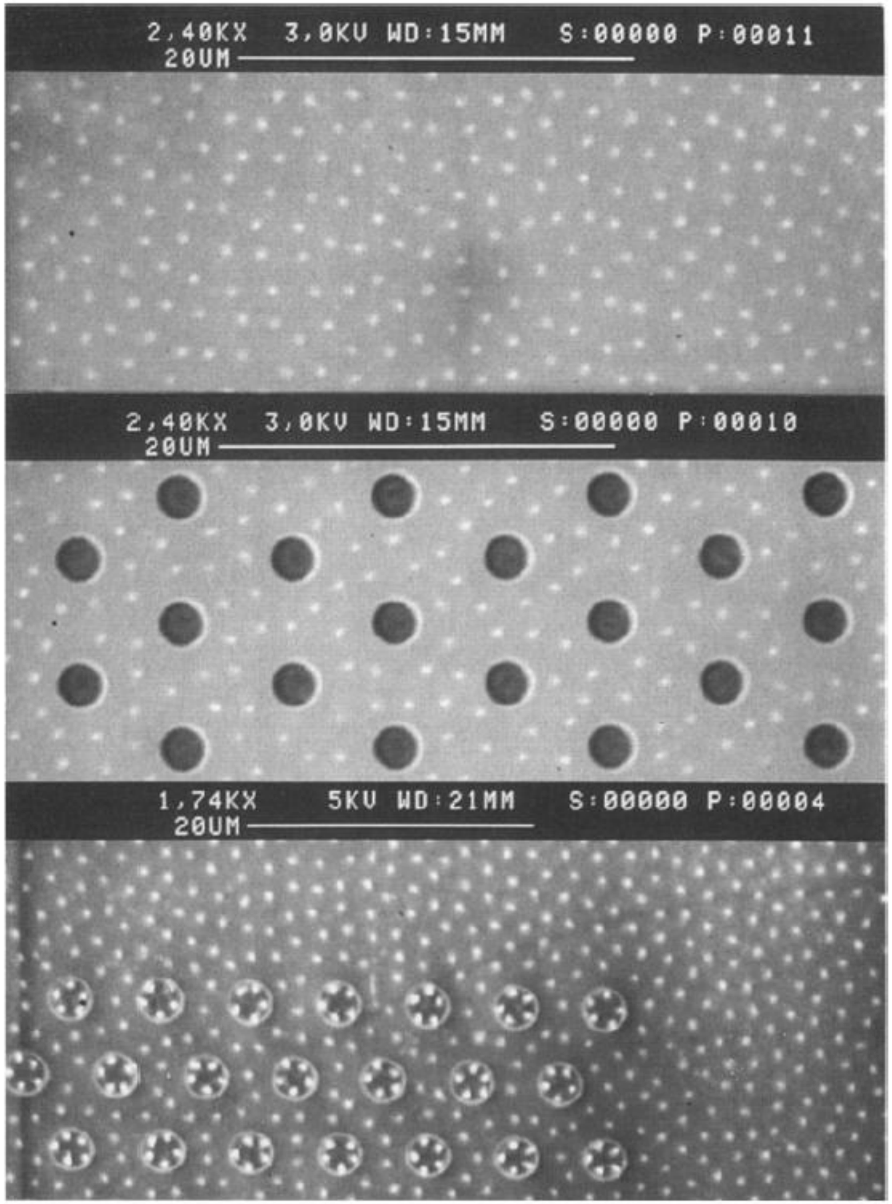
Magnetic field effect: Superconducting vortices



In superconductivity, a **fluxon** (also called an **Abrikosov vortex** or **quantum vortex, fluxons**) is a vortex of supercurrent in a type-II superconductors

https://blog.tmcnet.com/blog/tom-keating/technology-and-science/quantum-levitation-back-to-the-future-hoverboard.asp

Vortices in superconducting films with "through" and "blind" holes ("antidots")



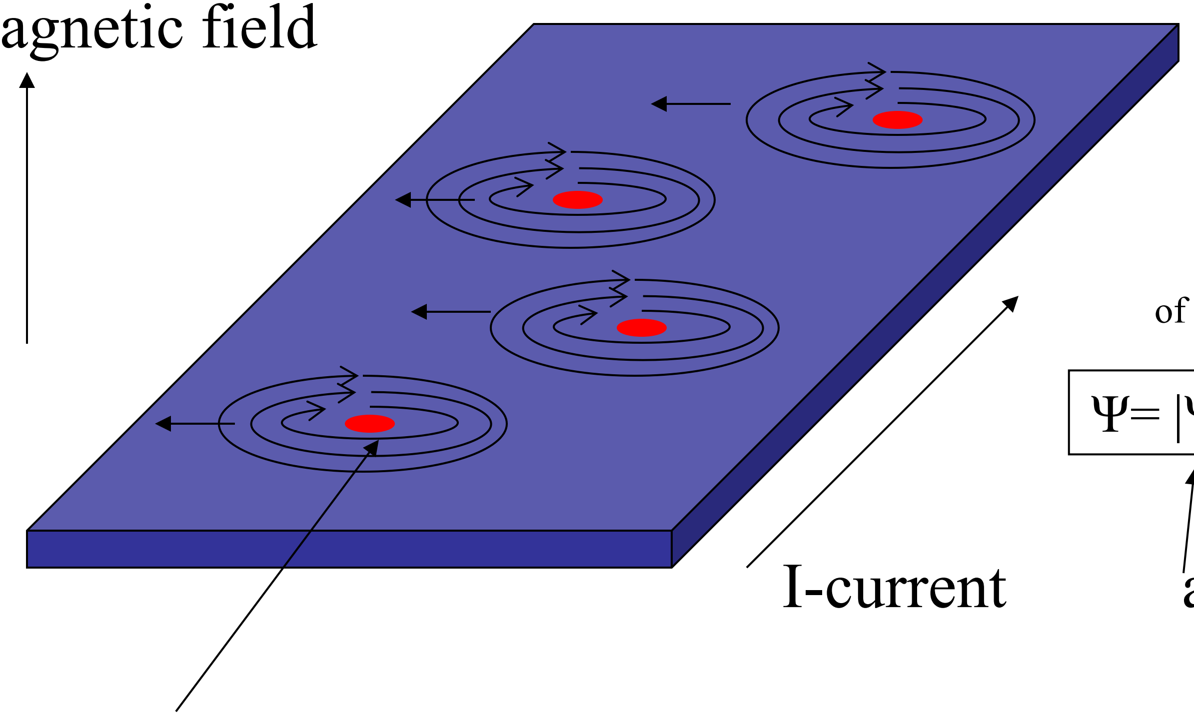
A. Bezryadin and B. Pannetier
"Role of Edge Superconducting States in Trapping of Multi-Quanta Vortices by Microholes. Application of the Bitter Decoration Technique",
J. of Low Temp. Phys., V.102, p.73 (1996).

Vortices are quantized tubes carrying magnetic field into superconductor

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance), if they move

B -magnetic field



I-current

Wave function
of all superconducting electrons:

$$\boxed{\Psi = |\Psi| \exp(i\varphi) = |\Psi| \exp(i\theta)}$$

amplitude

phase

Vortex core (red): normal, not superconducting; diameter $\xi \sim 10$ nm

The current is extended to a scale λ, which is larger than ξ in type II superconductors (such as thin films of any material)

Reminder: single electron in empty space

Wave function: $\Psi = |\Psi| \exp(ikx) = |\Psi| [\cos(kx) + i \sin(kx)]$

Wave number: $k=2\pi/\lambda$

$i*i=-1$

General form: $\Psi = |\Psi| \exp(-i\varphi)$

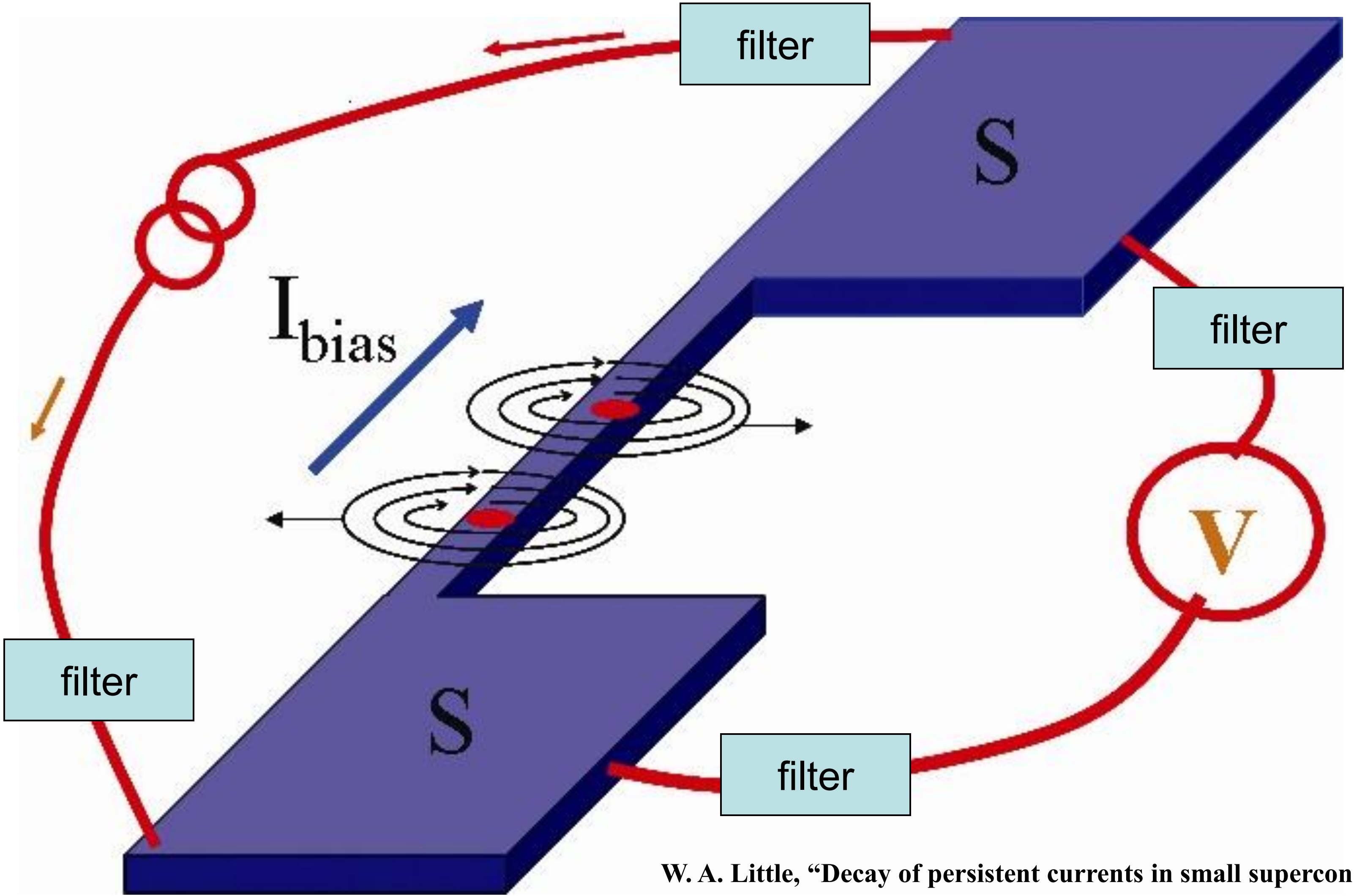
In this example of a plane wave, the phase is: $\varphi=kx$

Momentum: $p=(h/2\pi)k$

DC transport measurement schematic to detect passing vortices

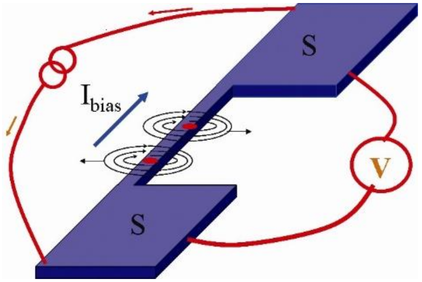
Bleu: superconducting film and wire

Red: Phase slip events or crossing vortices



W. A. Little, “Decay of persistent currents in small superconductors”, Physical Review, V.156, pp.396-403 (1967).

How to use voltage to determine the rate of phase slips?



Key principle: every time a vortex crosses the wire the phase difference changes by 2π.

Phase evolution equation: $d\varphi/dt = 2eV/\hbar$

Simplified derivation:

1. Time-dependent Schrödinger equation with fixed energy:

________________$i\hbar(d\Psi/dt)=E\,\Psi$

2. The solution is: $\Psi=\exp(-iEt/\hbar)$ (here E is the energy)

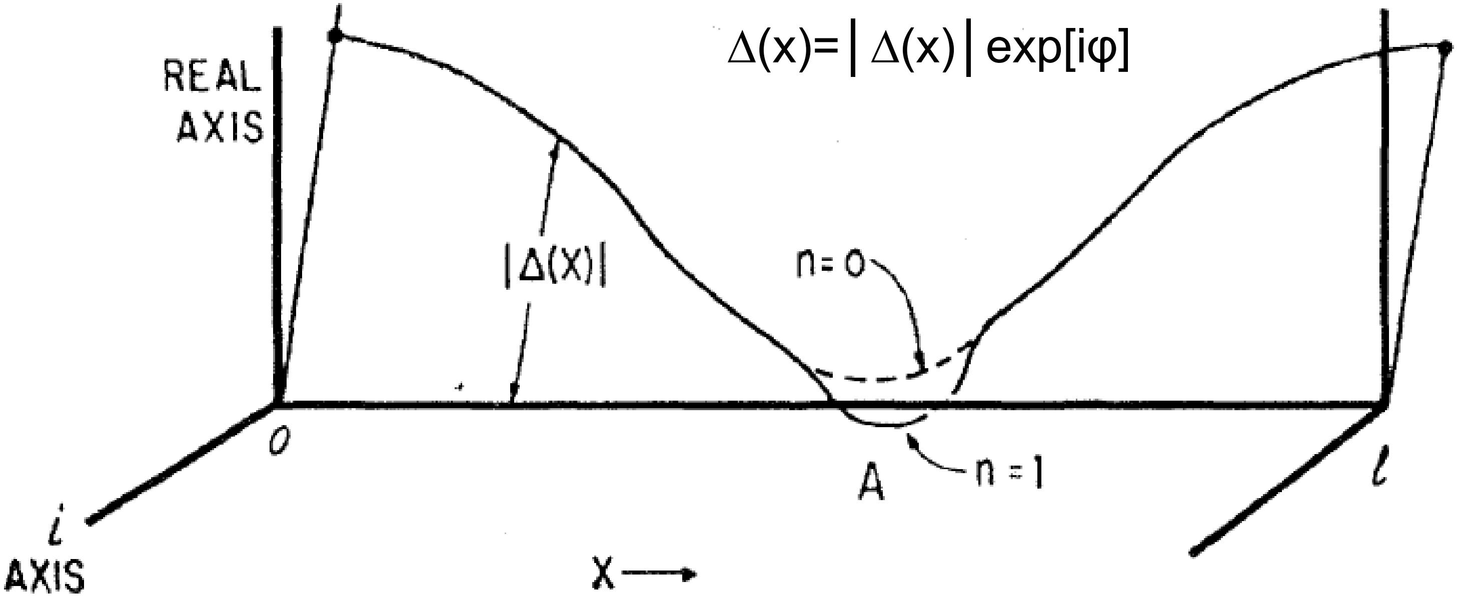
3. The phase of the wavefunction is $\varphi=Et/\hbar$

4. The energy is defined by the electric potential (voltage), V as follows: $E=2eV$. Note that the effective charge of superconducting electrons is 2e, where “e” is the charge of one electron. Such superconducting electron pairs are called Cooper pairs.

Thus, the resulting equation is: $d\varphi/dt = 2eV/\hbar$

Gor'kov, L.P. (1958) *Sov. Phys. JETP*, 7, 505.

Thin superconducting wire have some nonzero electrical resistance due to Little's Phase Slips



$$\Delta(x)=|\Delta(x)|\exp[i\varphi]$$

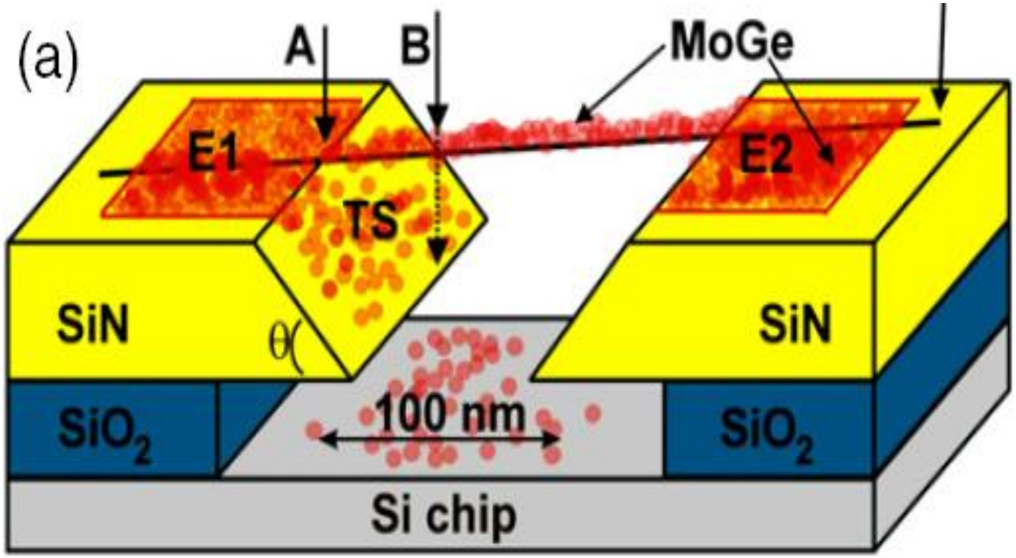
W. A. Little, "Decay of persistent currents in small superconductors", Physical Review, V.156, pp.396-403 (1967).

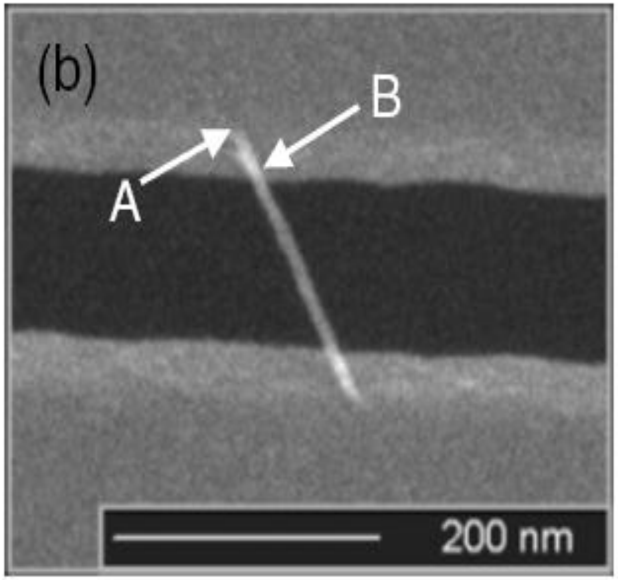
Two types of phase slips (PS) can occur:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Fabrication of nanowires

Method of Molecular Templating





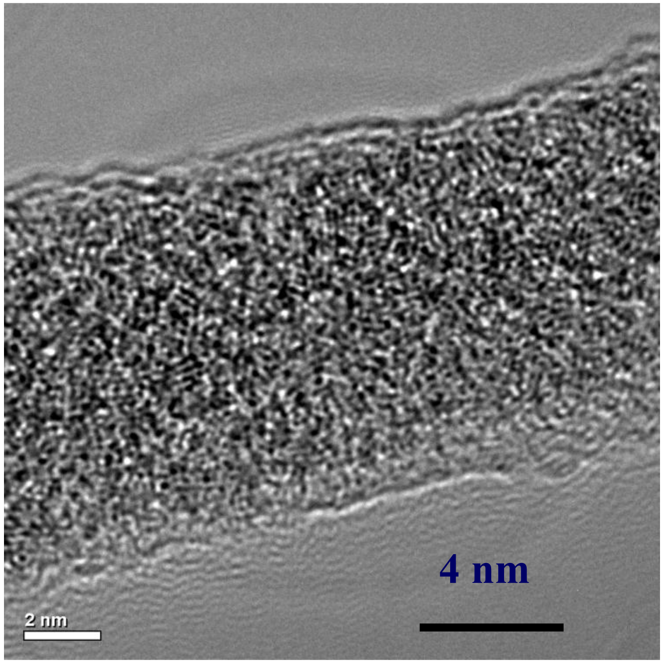
Si/ SiO_2/SiN substrate with undercut

~ 0.5 mm Si wafer
500 nm SiO_2
60 nm SiN
Width of the trenches ~ 50 - 500 nm

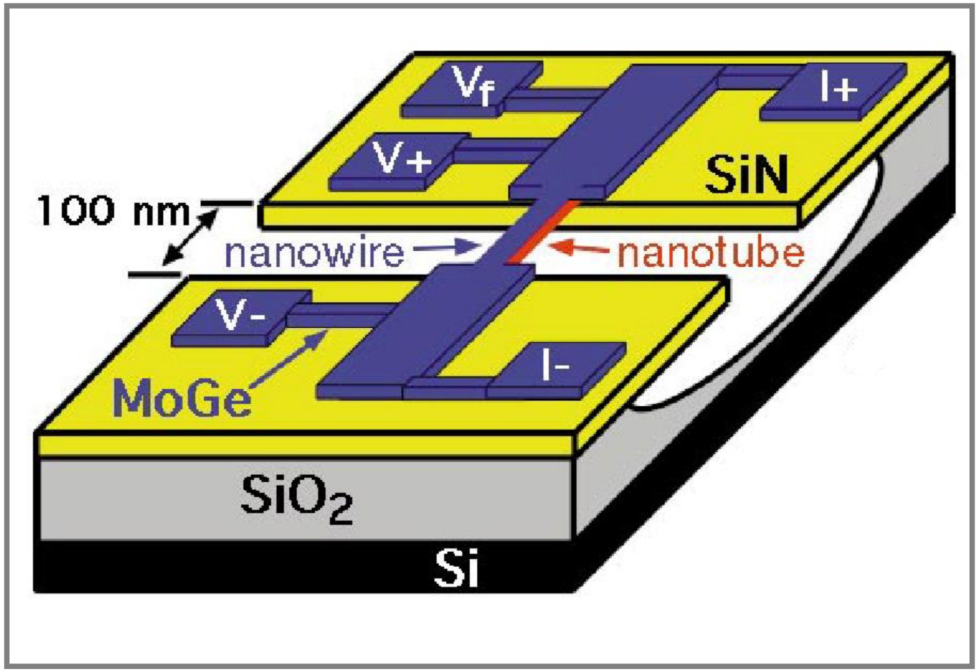
HF wet etch for ~10 seconds to form undercut

Bezryadin, Lau, Tinkham, *Nature* **404**, 971 (2000)

Sample Fabrication



TEM image of a wire shows amorphous wire morphology.
Nominal MoGe thickness = 3 nm



Schematic picture of the pattern Nanowire + Film Electrodes used in transport measurements

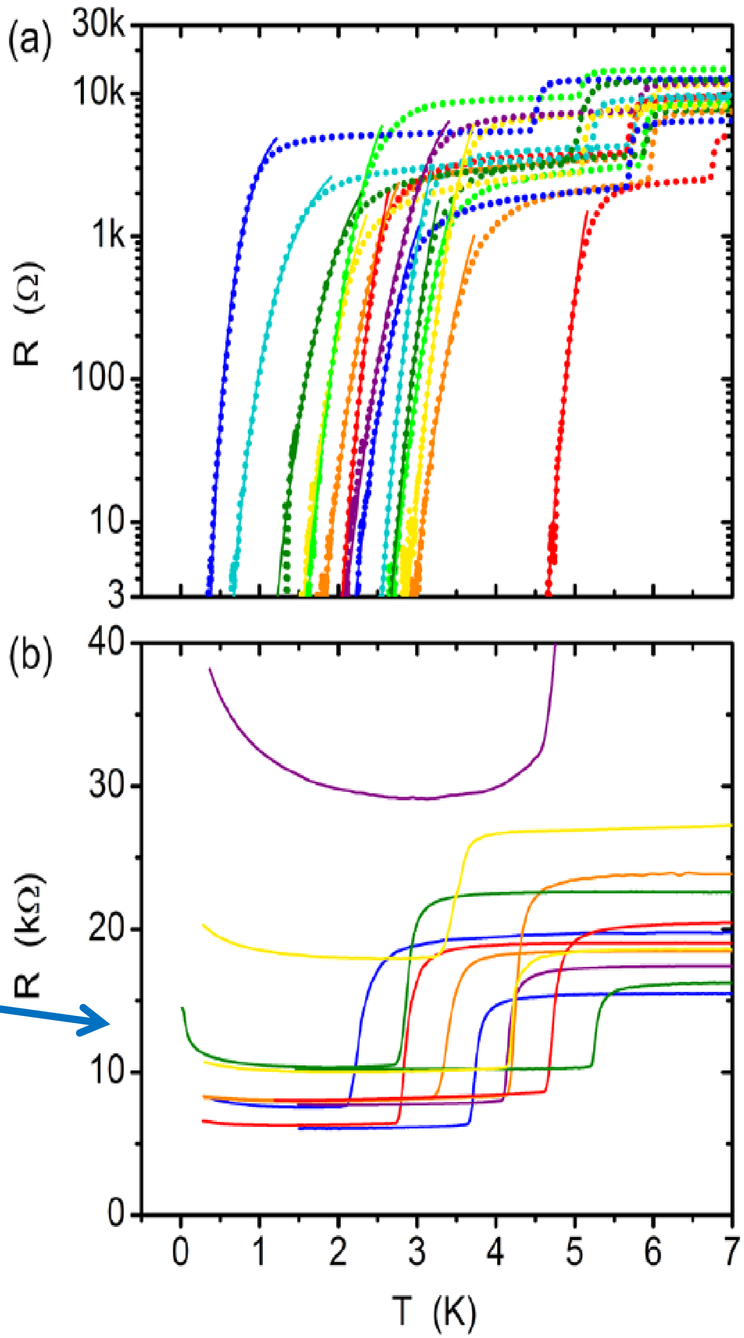
Dichotomy in nanowires: Evidence for superconductor-insulator transition (SIT)

R=V/I I~3 nA

The difference between samples is the amount of the deposited Mo79Ge21.

Thin wires become insulating if their normal resistance is lager than resistance quantum h/4/e/e=6.5KOhm

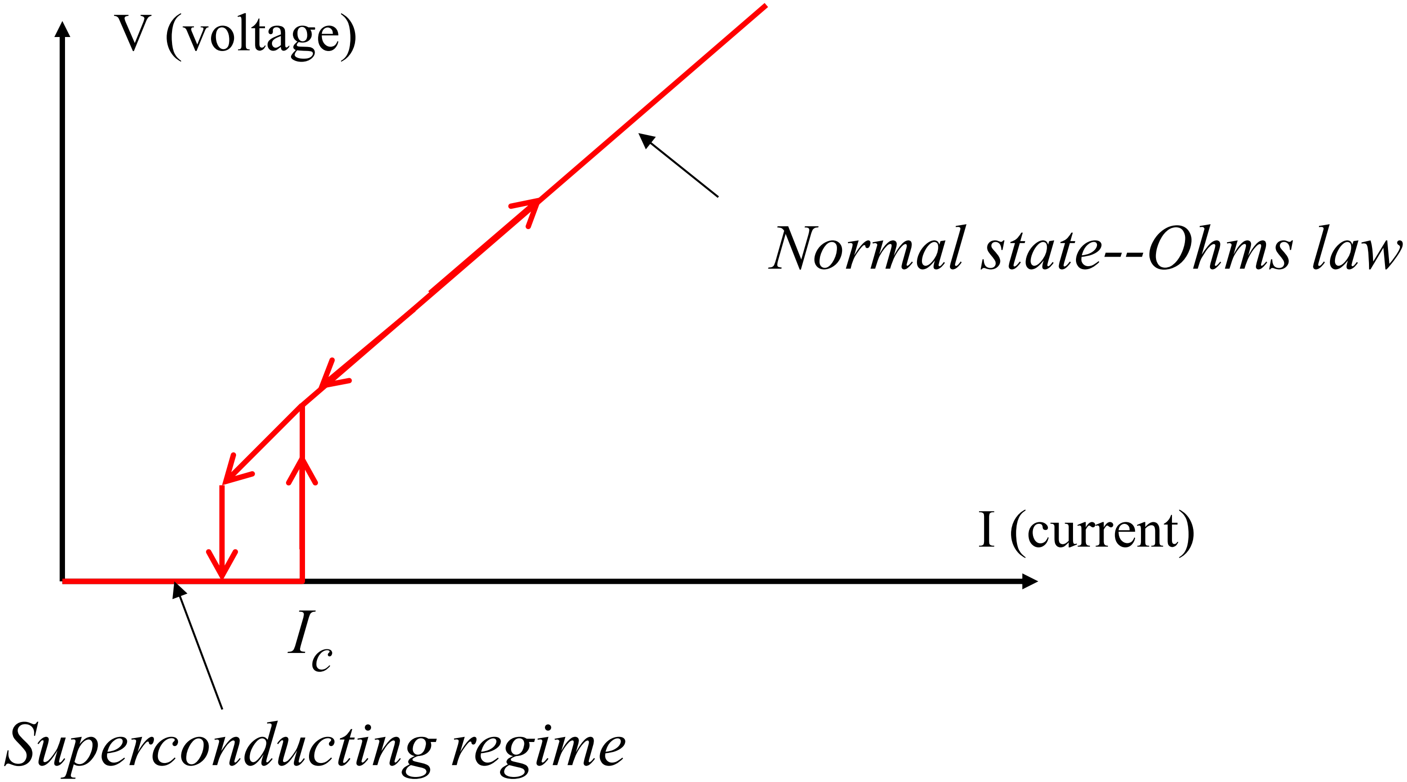
The insulating behavior is due to proliferation of quantum phase slips



Bollinger, Dinsmore, Rogachev, Bezryadin, Phys. Rev. Lett. **101**, 227003 (2008)

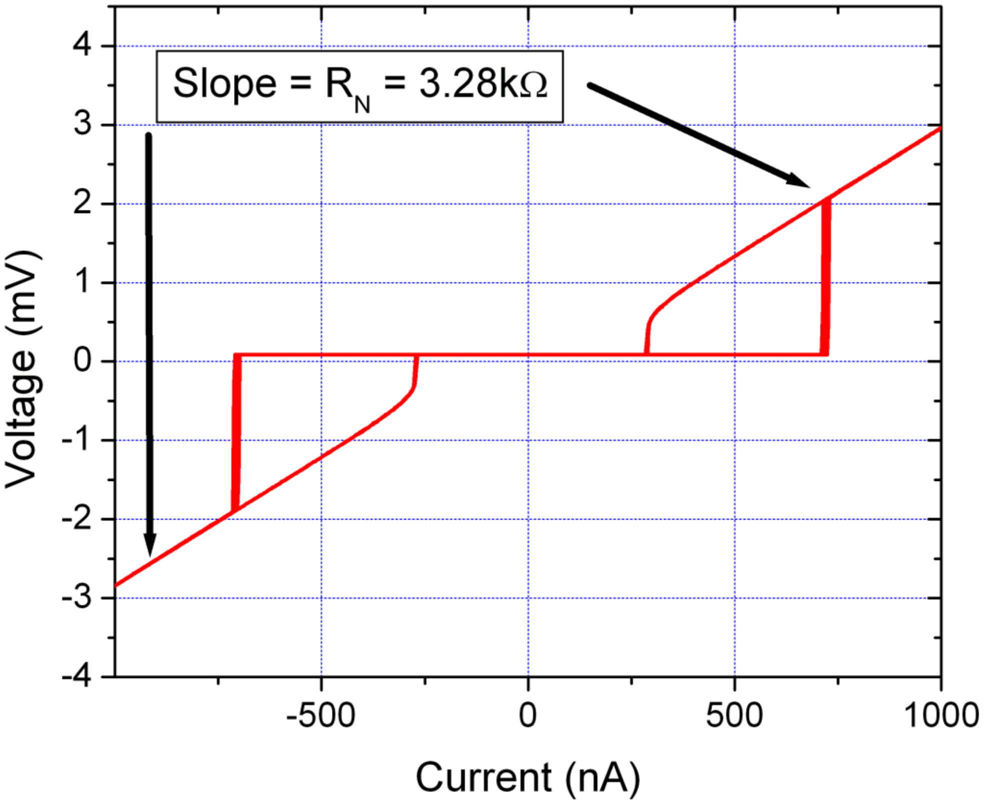
Expected voltage-current curve

Electrical resistance is zero only if current is not too strong

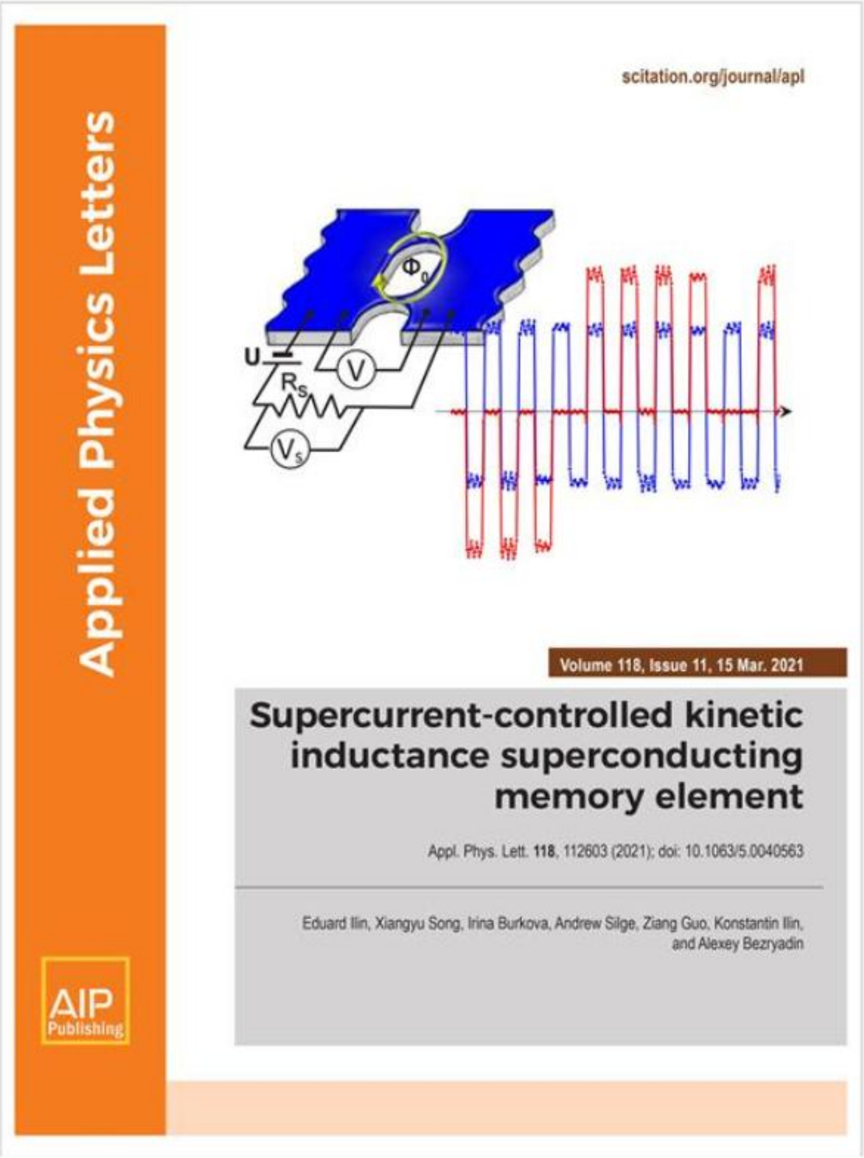


Experimental voltage-current curve.

Fluctuations of the switching current are due to Little's phase slips



Superconducting nanowire memory



scitation.org/journal/apl

Applied Physics Letters

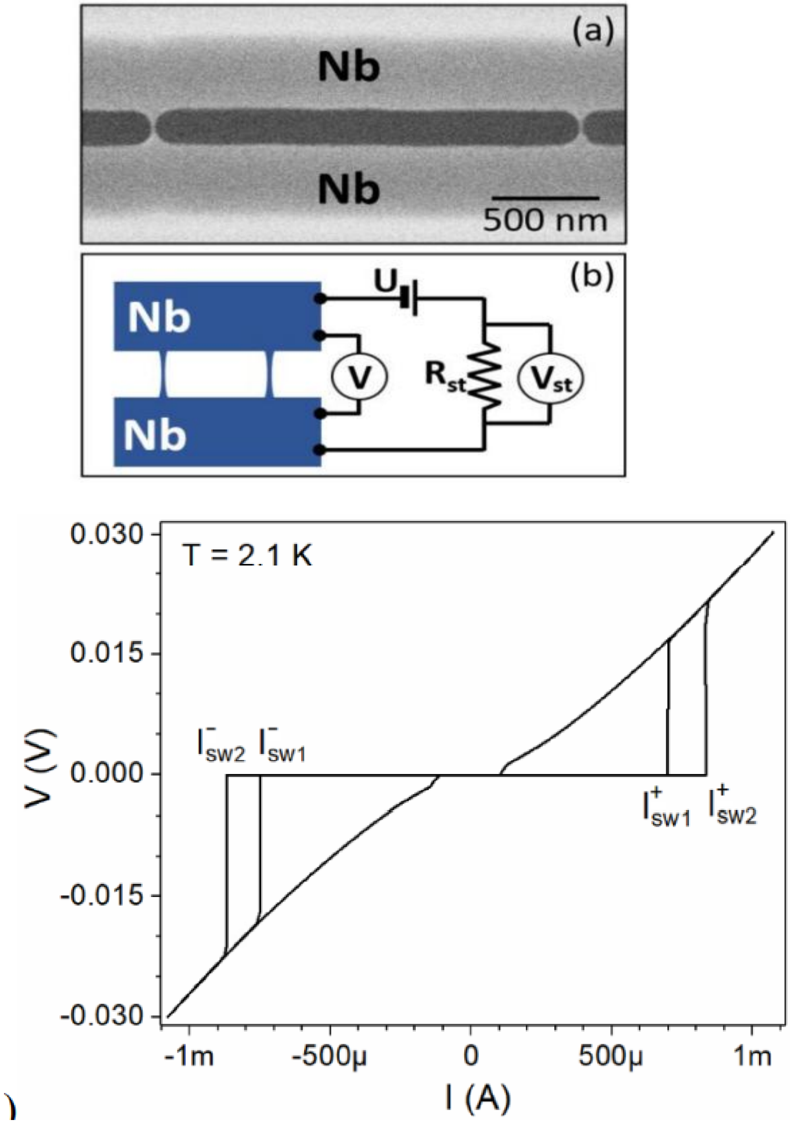
Volume 118, Issue 11, 15 Mar. 2021

Supercurrent-controlled kinetic inductance superconducting memory element

Appl. Phys. Lett. **118**, 112603 (2021); doi: 10.1063/5.0040563

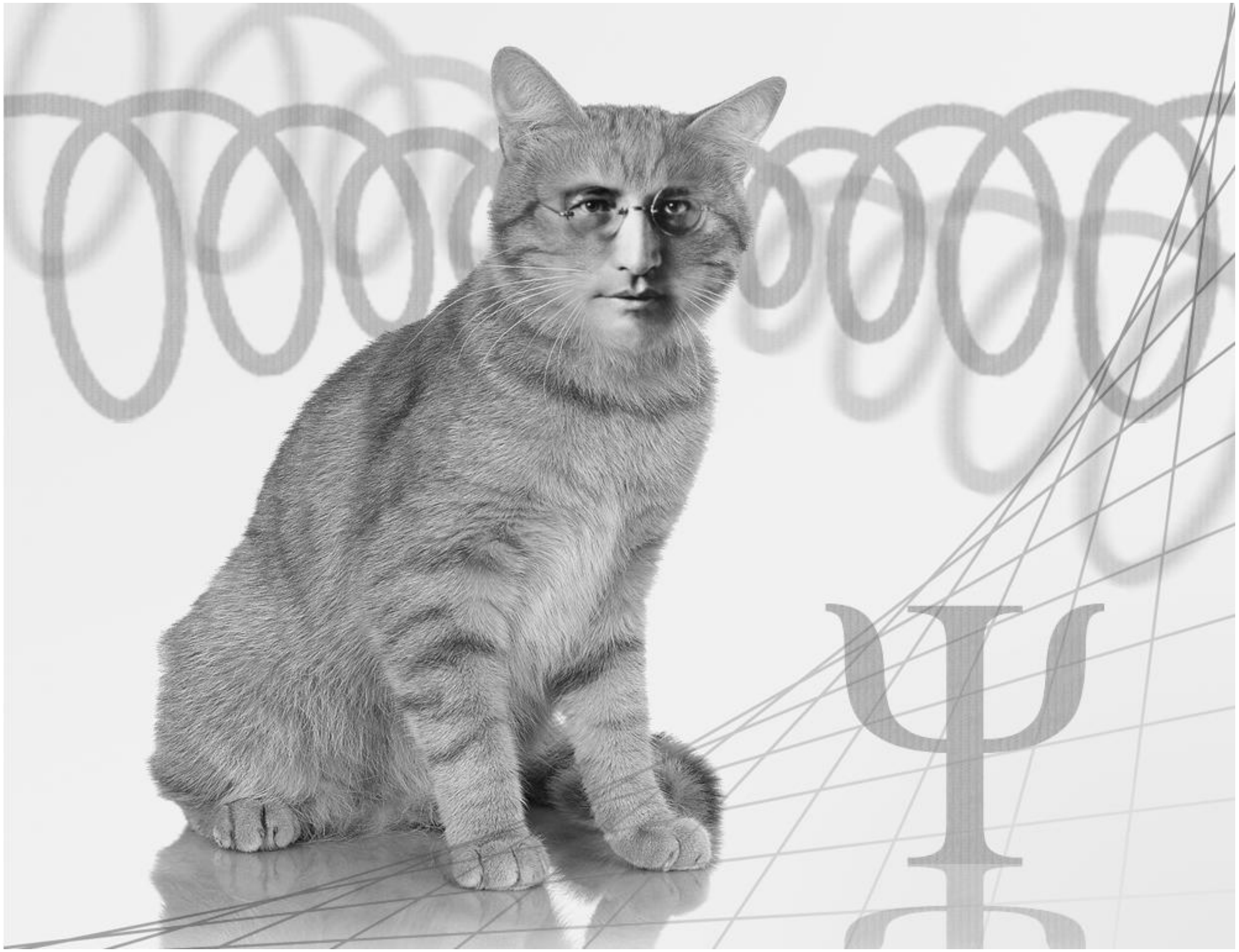
Eduard Ilin, Xiangyu Song, Irina Burkova, Andrew Silge, Ziang Guo, Konstantin Ilin, and Alexey Bezryadin

AIP Publishing

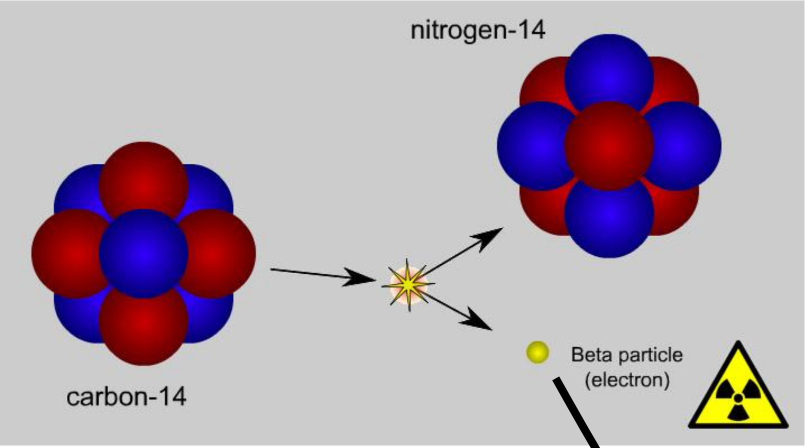


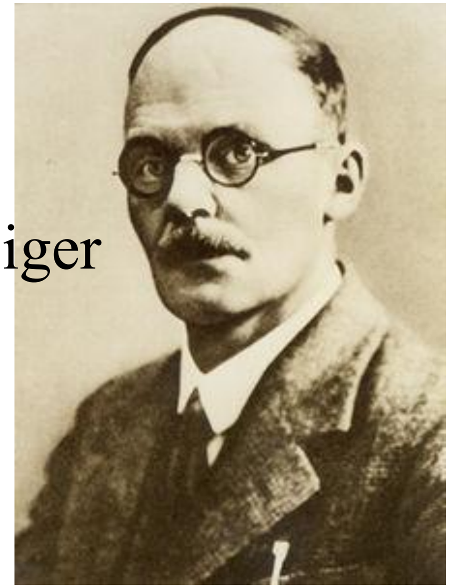
Schrödinger cat –
the ultimate macroscopic quantum phenomenon

E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment





Hans Geiger

Geiger counter

Linearity of the Schrödinger's equation

Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_1}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar\frac{\partial\psi_2}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

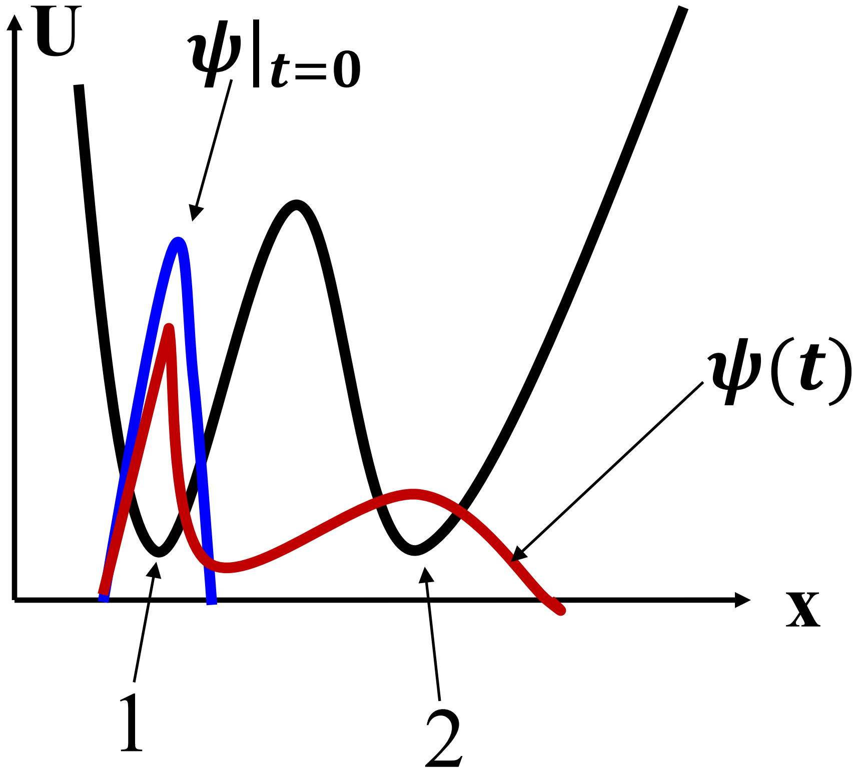
$$i\hbar\frac{\partial(\psi_1+\psi_2)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2(\psi_1+\psi_2)}{\partial x^2} + U(x)(\psi_1+\psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called "quantum superposition" of state (1) and (2)

Quantum tunneling

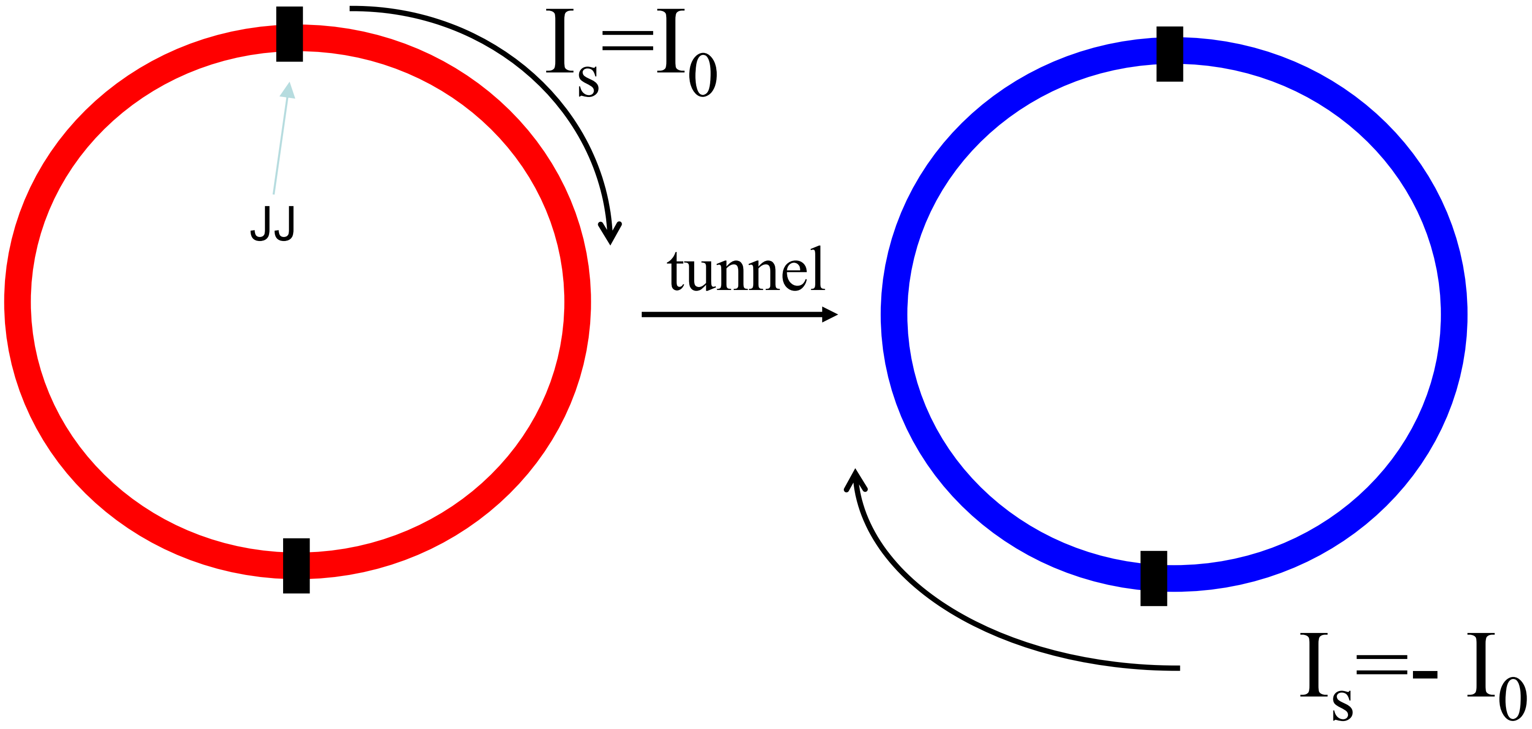
George Gamow

(He also helped to developed
Big Bang theory)



Quantum tunneling is possible since quantum superpositions of states are possible.

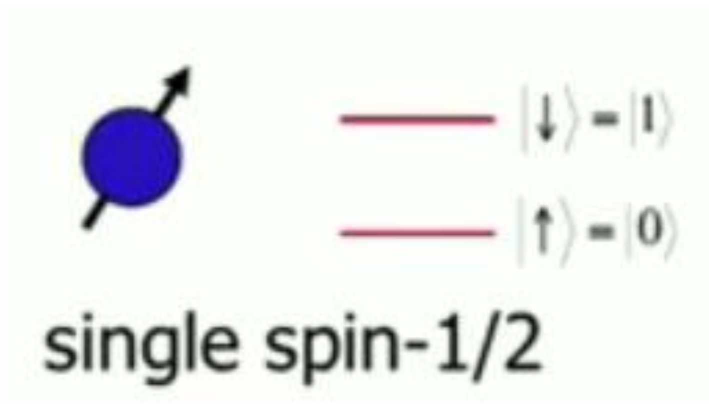
What sort of tunneling we will consider?

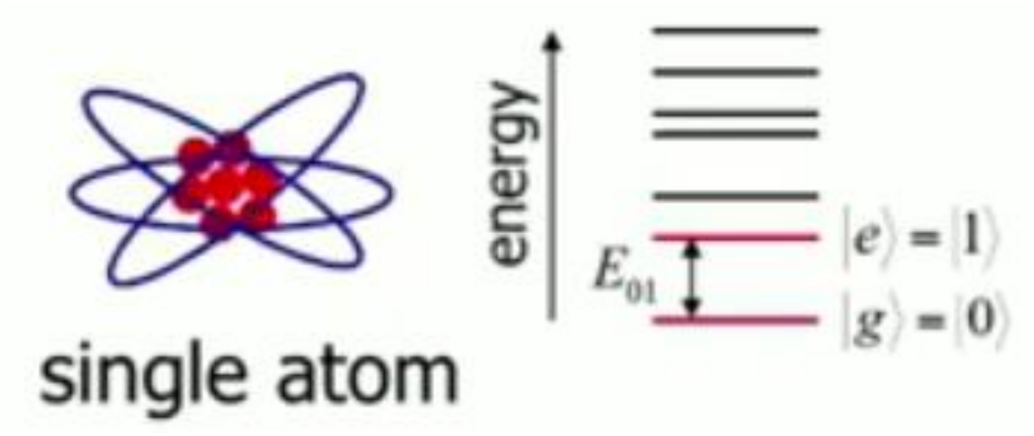


-Red color represents some strong current in the superconducting wire loop

-Blue color represents zero current in the loop

Types of Qubit



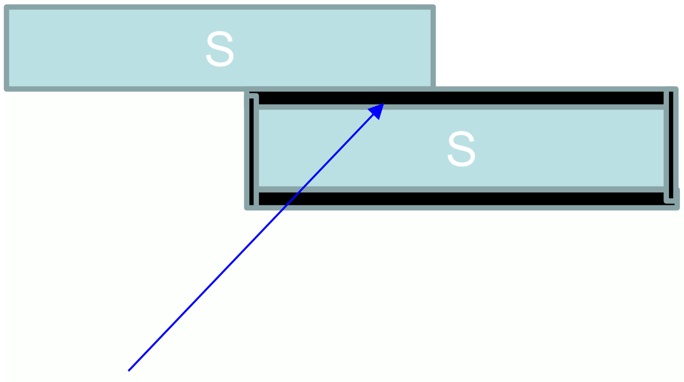


Quantum state:
$|\psi>=A*|0>+B*1>$

$A^2+B^2=1$

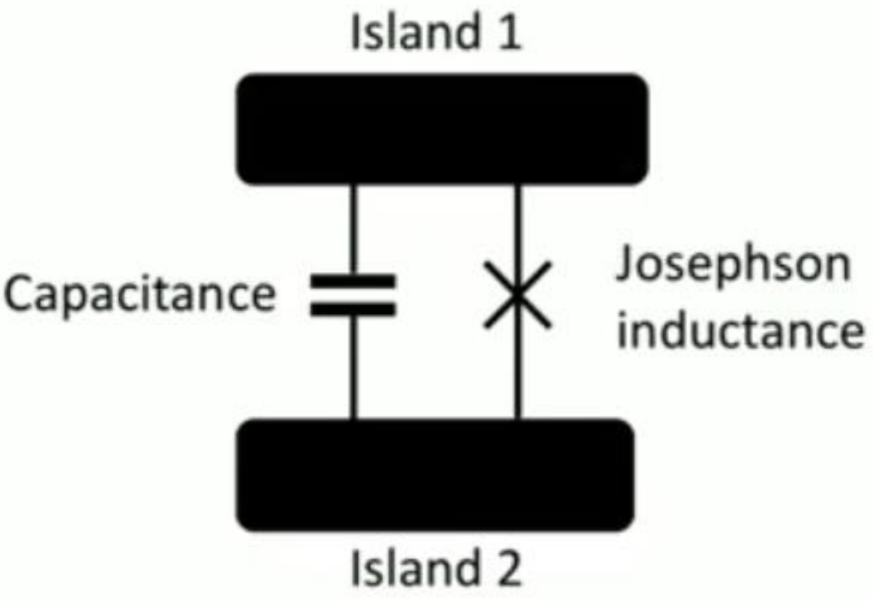
A and B are complex numbers

Transmon Qubit



Oxide: forms JJ

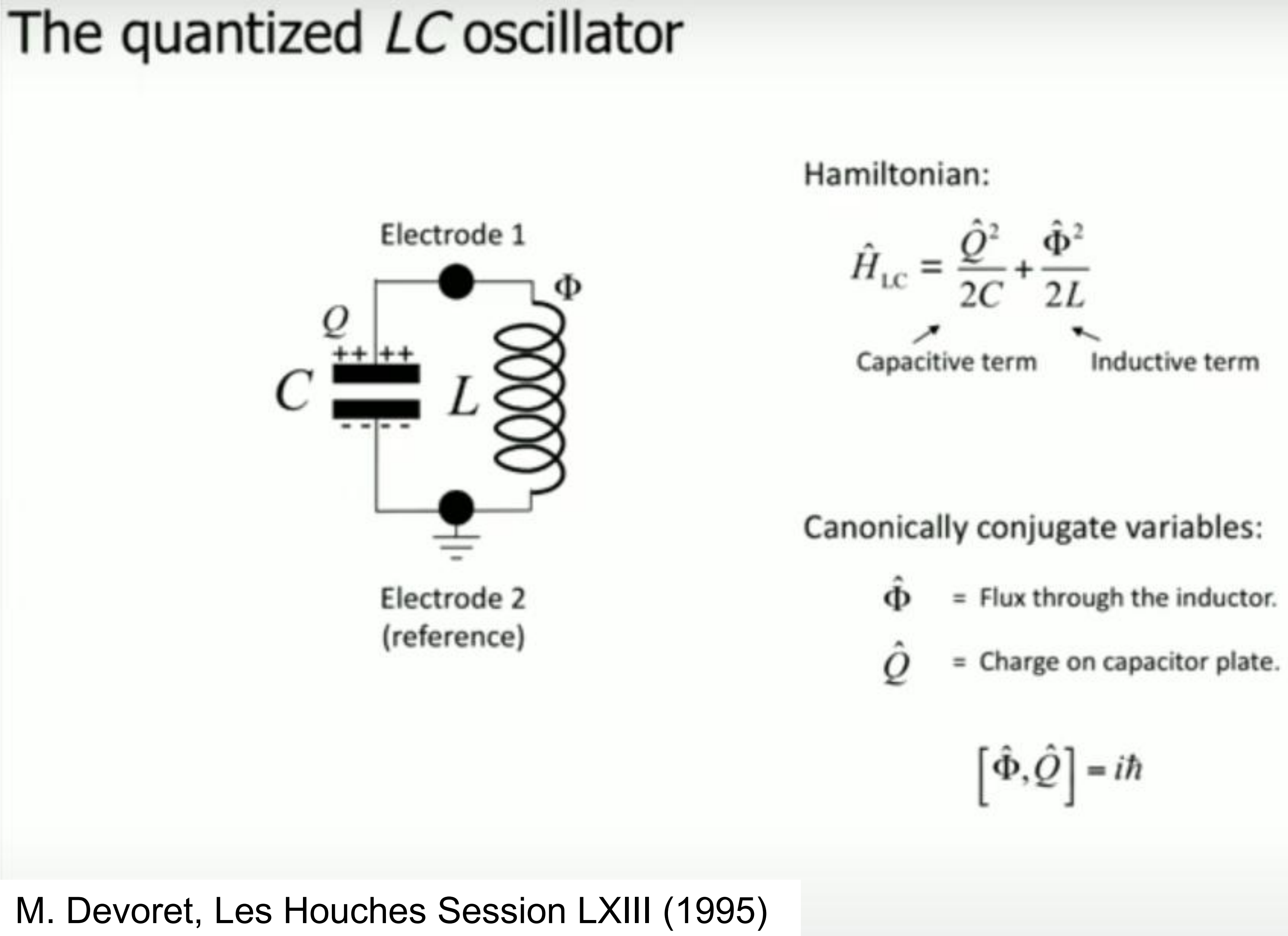
Electrical schematic of qubit



Theory of the transmon: J. Koch *et al.*, Phys Rev. A **76**, 042319 (2007)

Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits



The quantized *LC* oscillator

Hamiltonian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term

Inductive term

Canonically conjugate variables:

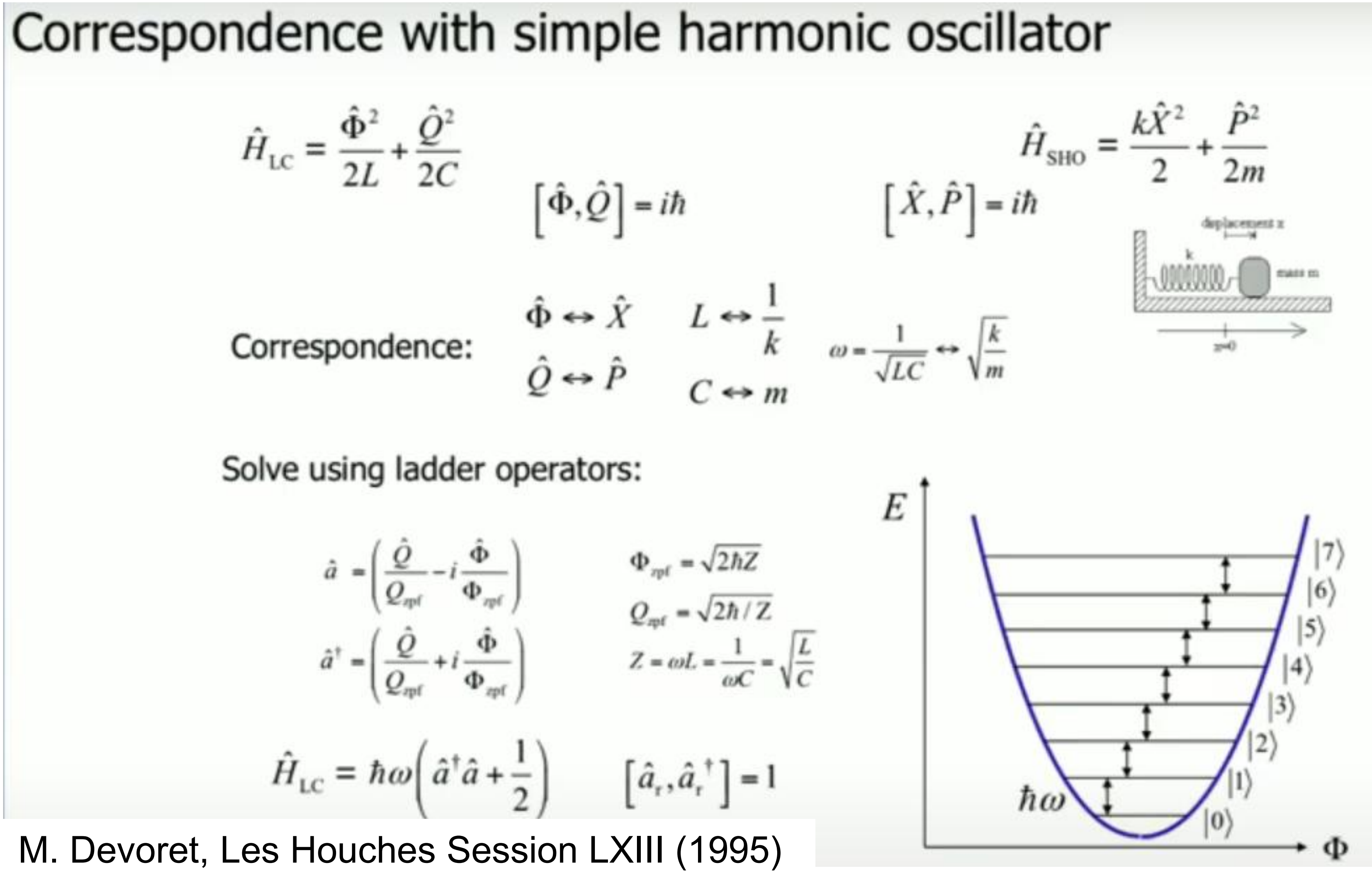
$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

M. Devoret, Les Houches Session LXIII (1995)

Discrete energy spectrum of the LC-circuit



Correspondence with simple harmonic oscillator

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

$$\hat{H}_{\mathrm{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar \qquad \left[\hat{X}, \hat{P}\right] = i\hbar$$

Correspondence:

$$\hat{\Phi} \leftrightarrow \hat{X} \qquad L \leftrightarrow \frac{1}{k}$$

$$\hat{Q} \leftrightarrow \hat{P} \qquad C \leftrightarrow m$$

$$\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$

Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{zpf}} - i\frac{\hat{\Phi}}{\Phi_{zpf}}\right)$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{zpf}} + i\frac{\hat{\Phi}}{\Phi_{zpf}}\right)$$

$$\Phi_{zpf} = \sqrt{2\hbar Z}$$

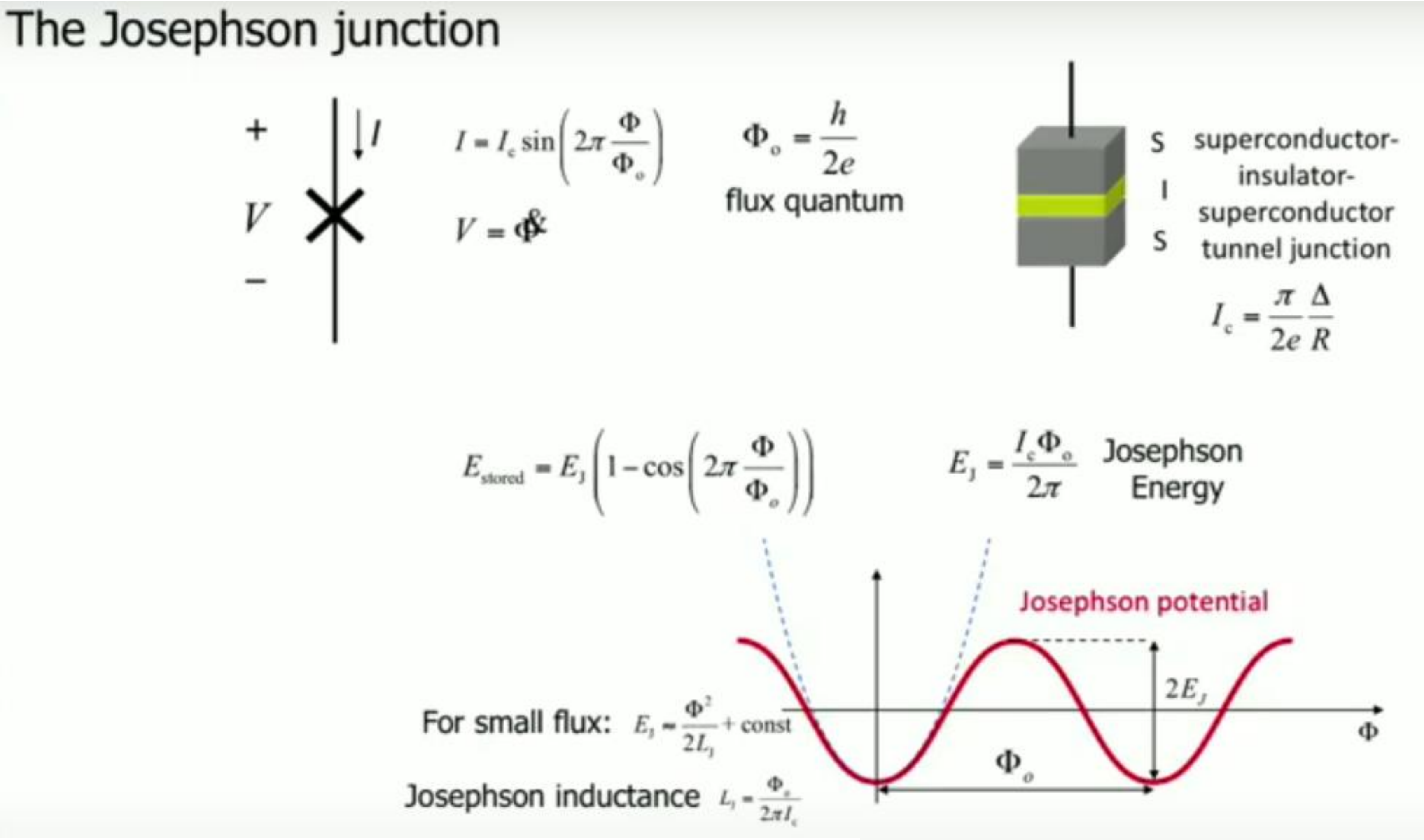
$$Q_{zpf} = \sqrt{2\hbar / Z}$$

$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\mathrm{LC}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \left[\hat{a}_r, \hat{a}_r^{\dagger}\right] = 1$$

M. Devoret, Les Houches Session LXIII (1995)

Non-harmonicity is the key factor



The Josephson junction

$I = I_c \sin\left(2\pi\frac{\Phi}{\Phi_o}\right)$

$V = \dot{\Phi}$

$\Phi_o = \frac{h}{2e}$

flux quantum

superconductor-insulator-superconductor tunnel junction

$I_c = \frac{\pi}{2e}\frac{\Delta}{R}$

$E_{\text{stored}} = E_J\left(1-\cos\left(2\pi\frac{\Phi}{\Phi_o}\right)\right)$

$E_J = \frac{I_c\Phi_o}{2\pi}$ Josephson Energy

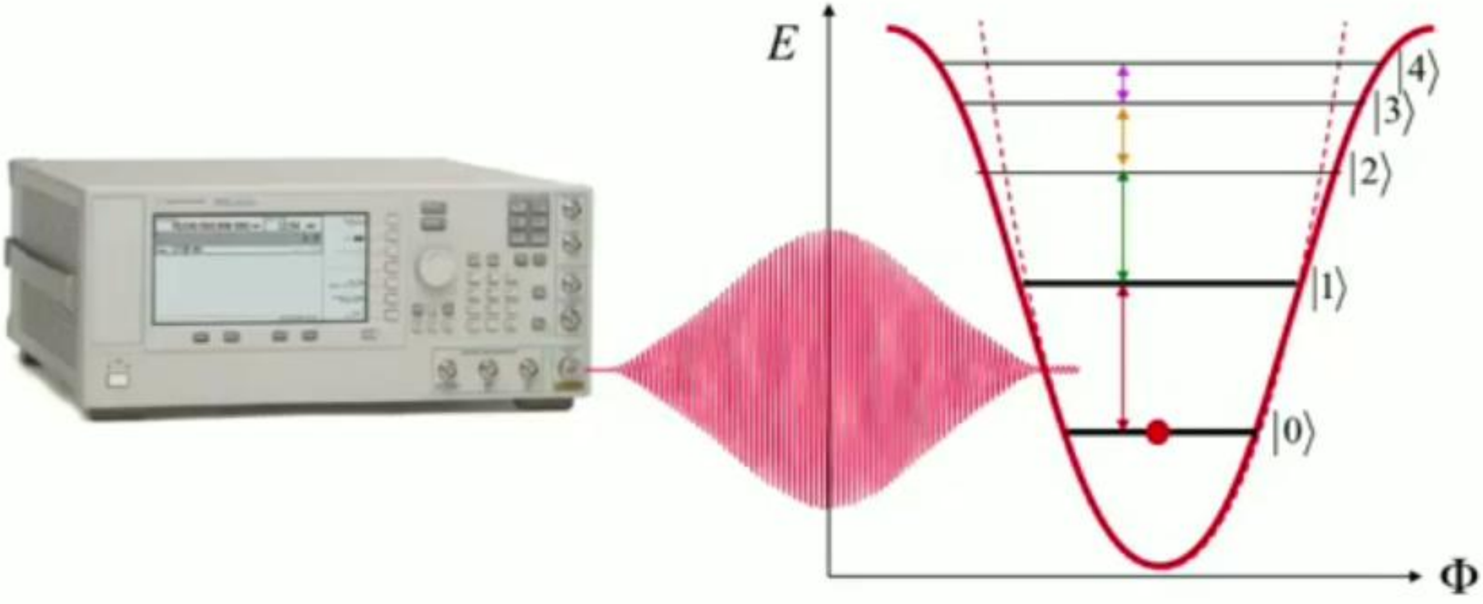
For small flux: $E_J = \frac{\Phi^2}{2L_J} + \text{const}$

Josephson inductance $L_J = \frac{\Phi_o}{2\pi I_c}$

M. Devoret, Les Houches Session LXIII (1995)

Non-harmonicity is the key factor

Transmon energy spectrum



Meisnerron-transmon qubit

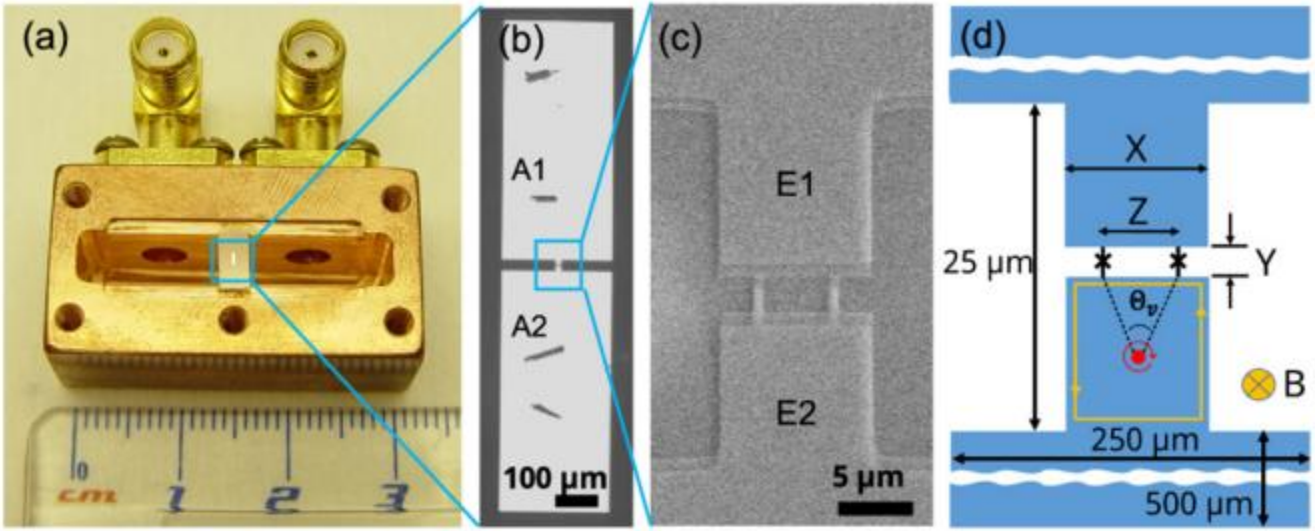


FIG. 1. (a) Optical image of the Meissner transmon qubit fabricated on a sapphire chip, which is mounted in the copper cavity. (b) A zoomed-in optical image of the qubit. Two rectangular pads marked A1 and A2 act as an RF antenna and shunt capacitor. (c) Scanning electron microscope (SEM) image of the electrodes marked E1 and E2, and a pair of JJs. (d) Schematics of the Meissner qubit. The X, Y, and Z denote the width, the distance between the electrodes, and the distance between two JJs, which are indicated by × symbols. The red dot and circular arrow around it in the bottom electrode represent a vortex and vortex current flowing clockwise, respectively. Θ_v is a polar angle defined by two dashed lines connecting the vortex and two JJs. The orange rectangular loop on the boundary of the bottom electrode indicates the Meissner current circulating counterclockwise.

J. Ku et al., PHYSICAL REVIEW B 94, 165128 (2016)

Meisnerron-transmon qubit

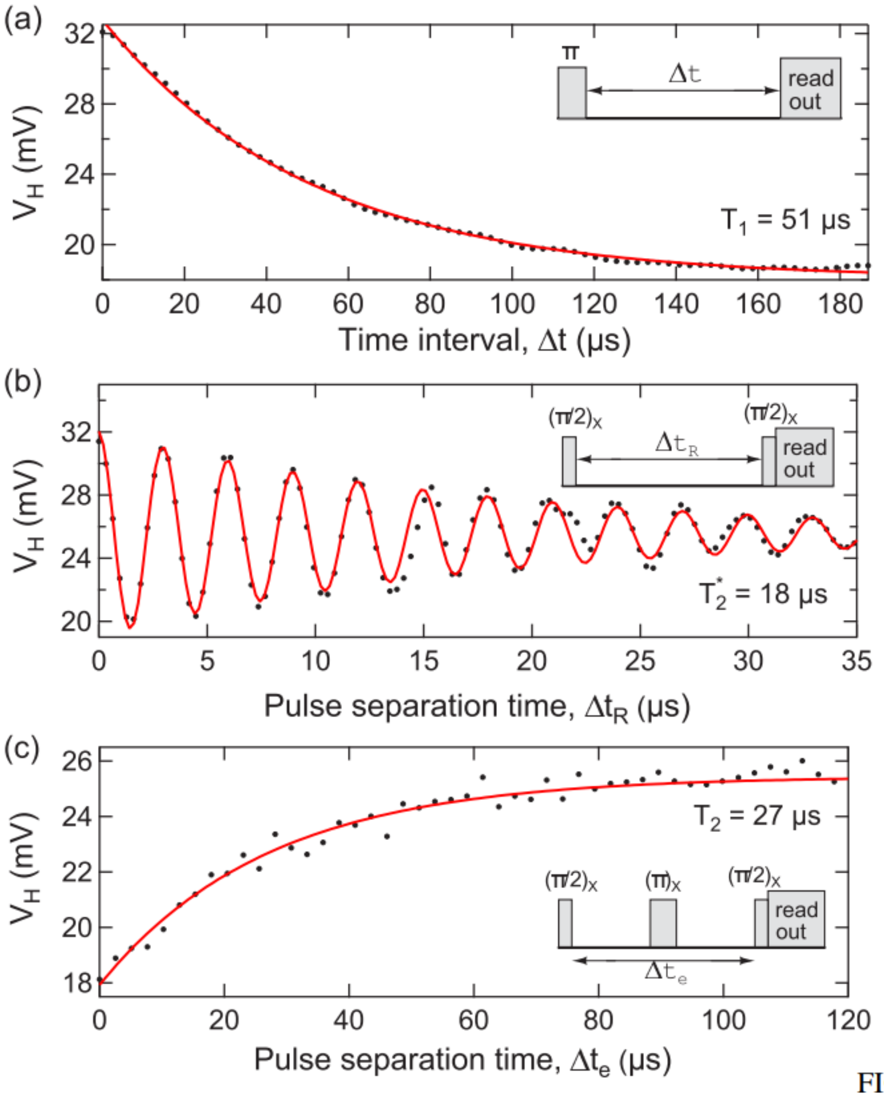


FIG. 4. Time domain measurements of the N7 sample at $B = 7.5$ mG. (a) Relaxation time measurement ($T_1 = 51\ \mu$s). (b) Ramsey fringe experiment ($T_2^* = 18\ \mu$s). (c) Hahn spin echo experiment ($T_2 = 27\ \mu$s). The red solid lines are the fits to the data. See the main text for the fitting functions.

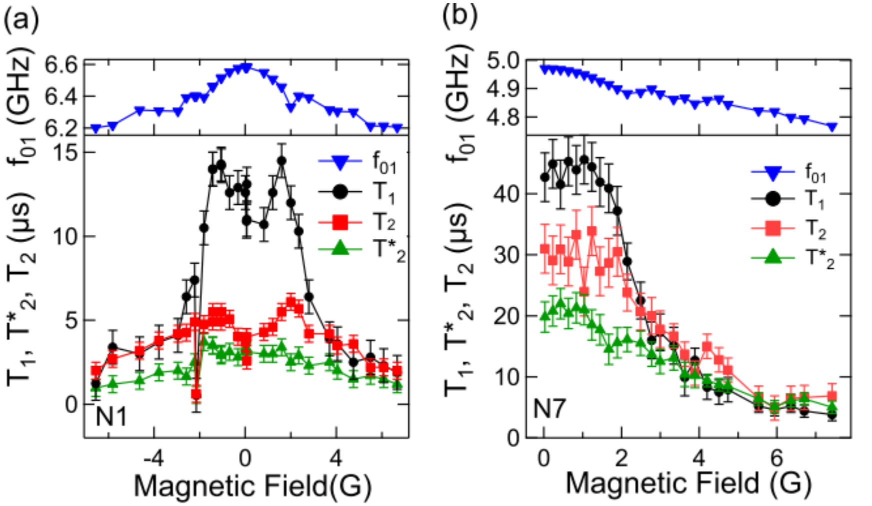


FIG. 6. The qubit transition frequencies (f_{01}) and three times scales (T_1, T_2^*, and T_2) were measured at the sweet spots over the wide range of magnetic field for the N1 (a) and N7 (b).

J. Ku et al., PHYSICAL REVIEW B 94, 165128 (2016)

Conclusions

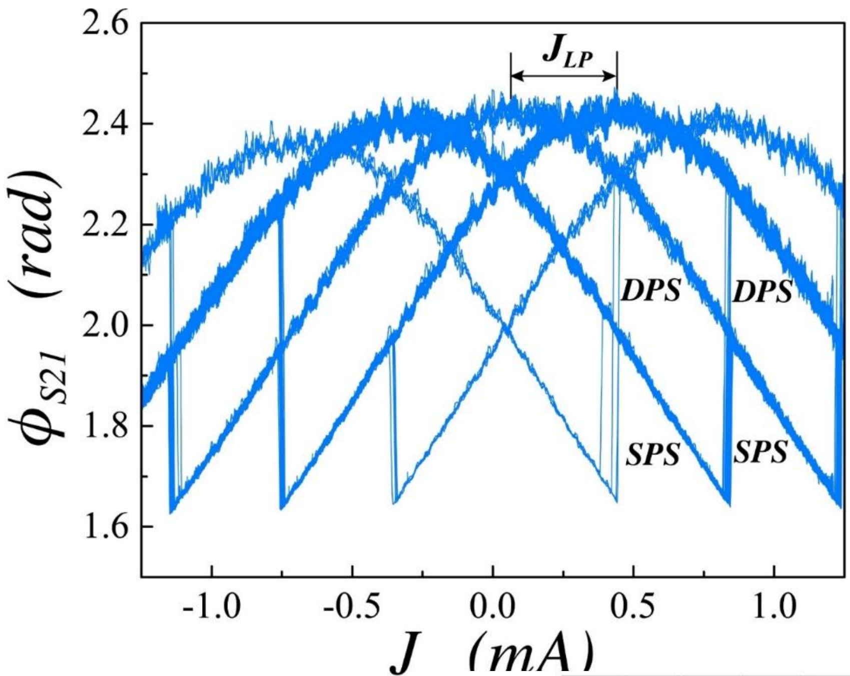
- Superconductivity is related to fundamental quantum phenomena. We have revied some of them. They will be discussed in more details in the future lectures.

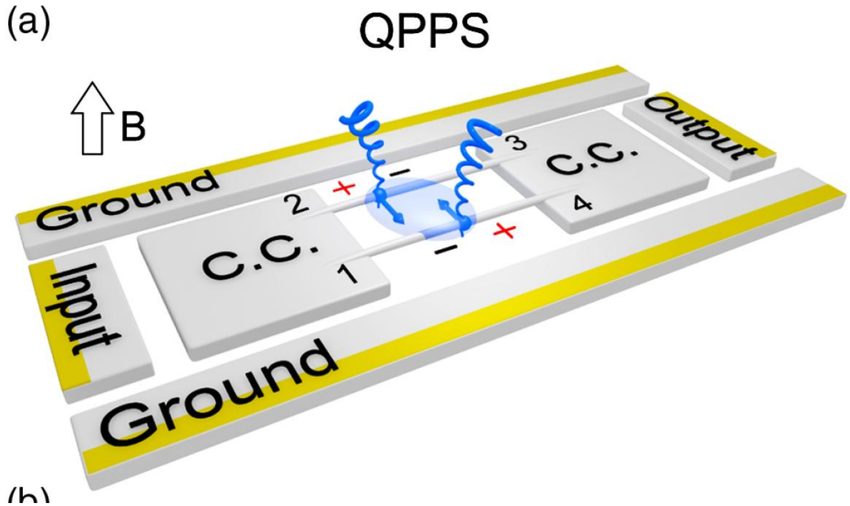
- Superconductors have been used to create strong and stable magnetic fields, in levitating trains for example.

- Superconducting quantum interference devices enabled researches to measure very small magnetic fields, such as those produced by human brain.

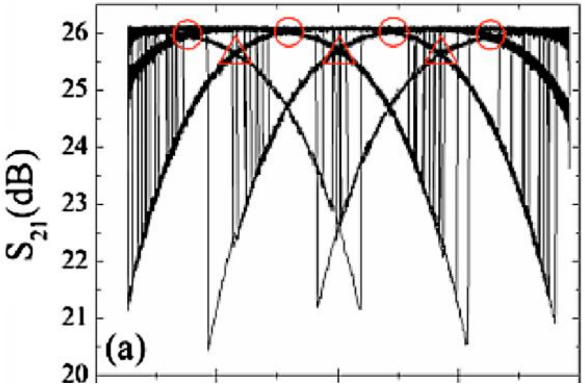
- Superconductors are used to build qubits, which are the building blocks of quantum computers.

Superconducting nanowire memory: microwave readout





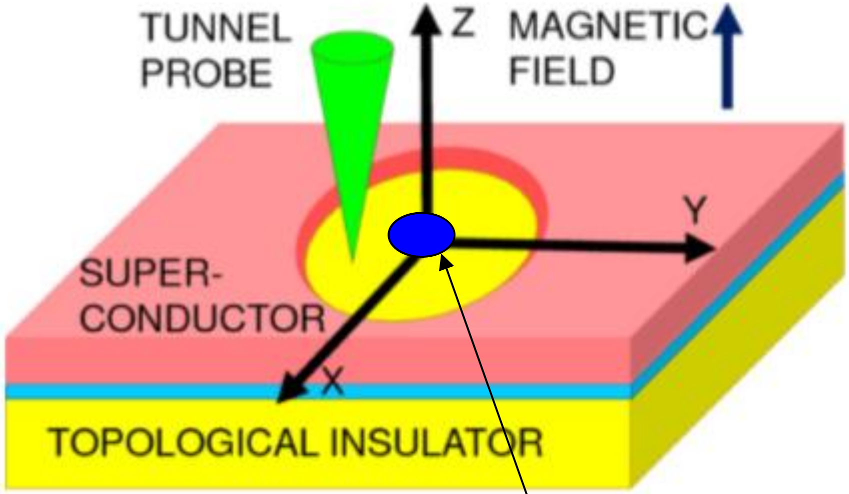
A. Belkin et al, PRX **5**, 021023 (2015)



Andrey Belkin,[a] Matthew Brenner, Thomas Aref, Jaseung Ku, and Alexey Bezryadin,
PPLIED PHYSICS LETTERS **98**, 242504 (2011)

T = 360 mK
$f = f_0(H=0)$

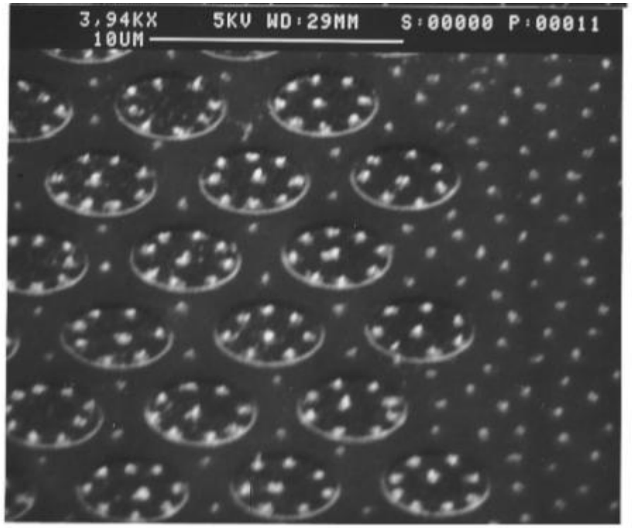
Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about Tc)

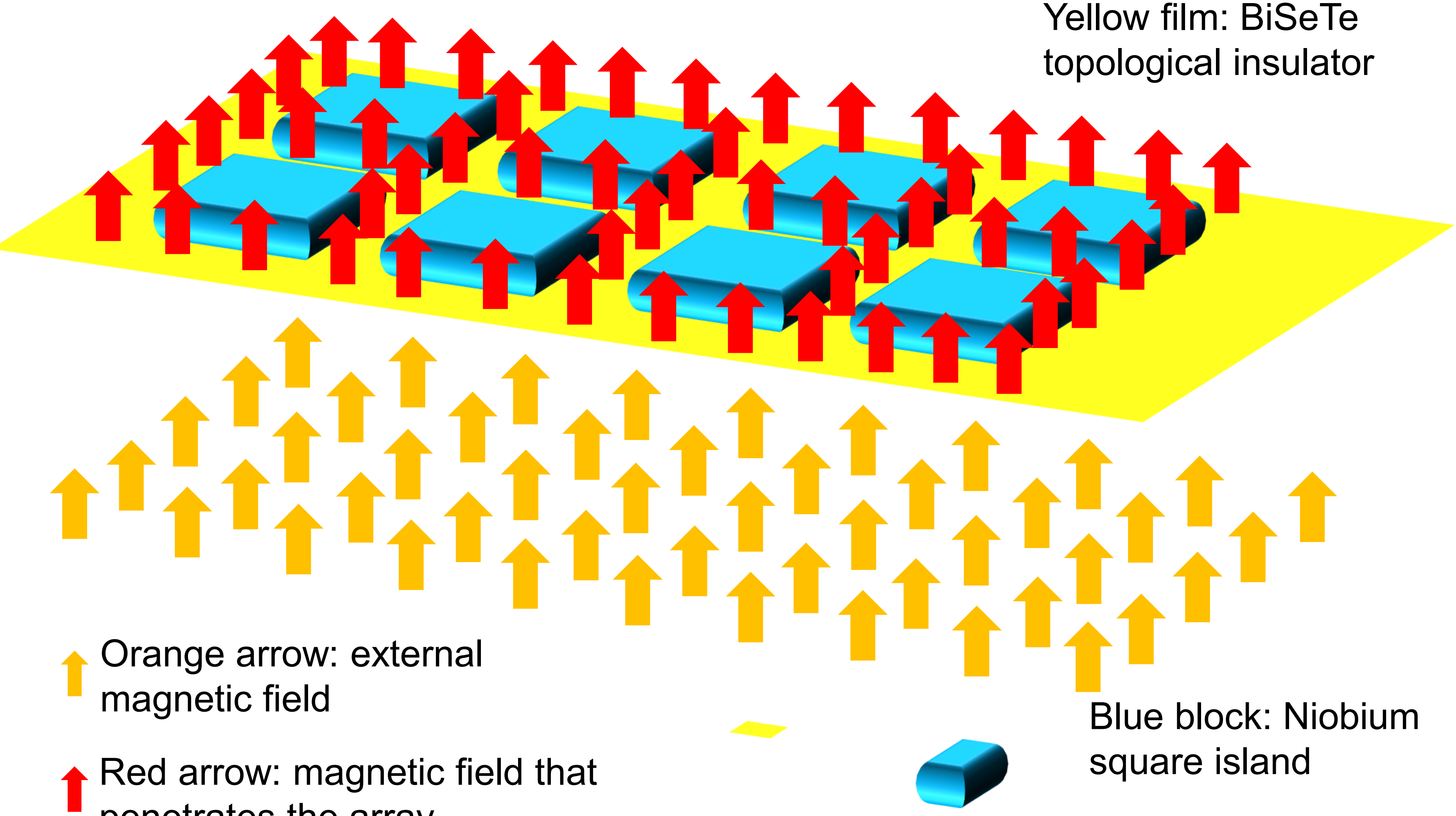
R.S. Akzyanov, A.V. Rozhkov, A.L.Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B 84, 075141 (2011)



A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

Superconducting array based on topological insulator- BST



Yellow film: BiSeTe topological insulator

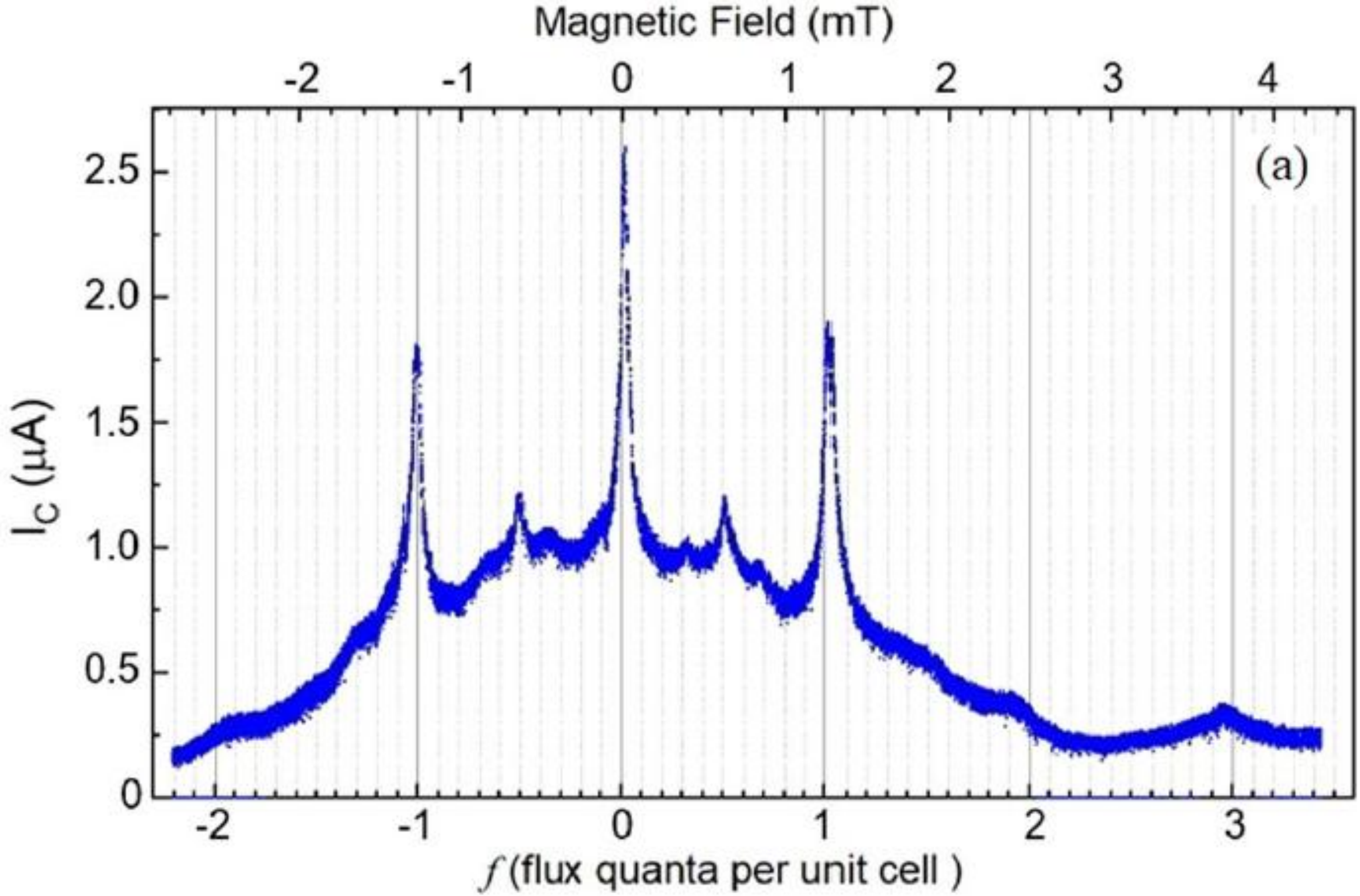
Orange arrow: external magnetic field

Red arrow: magnetic field that penetrates the array

Blue block: Niobium square island

Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Diffraction grating analogy with superconducting arrays



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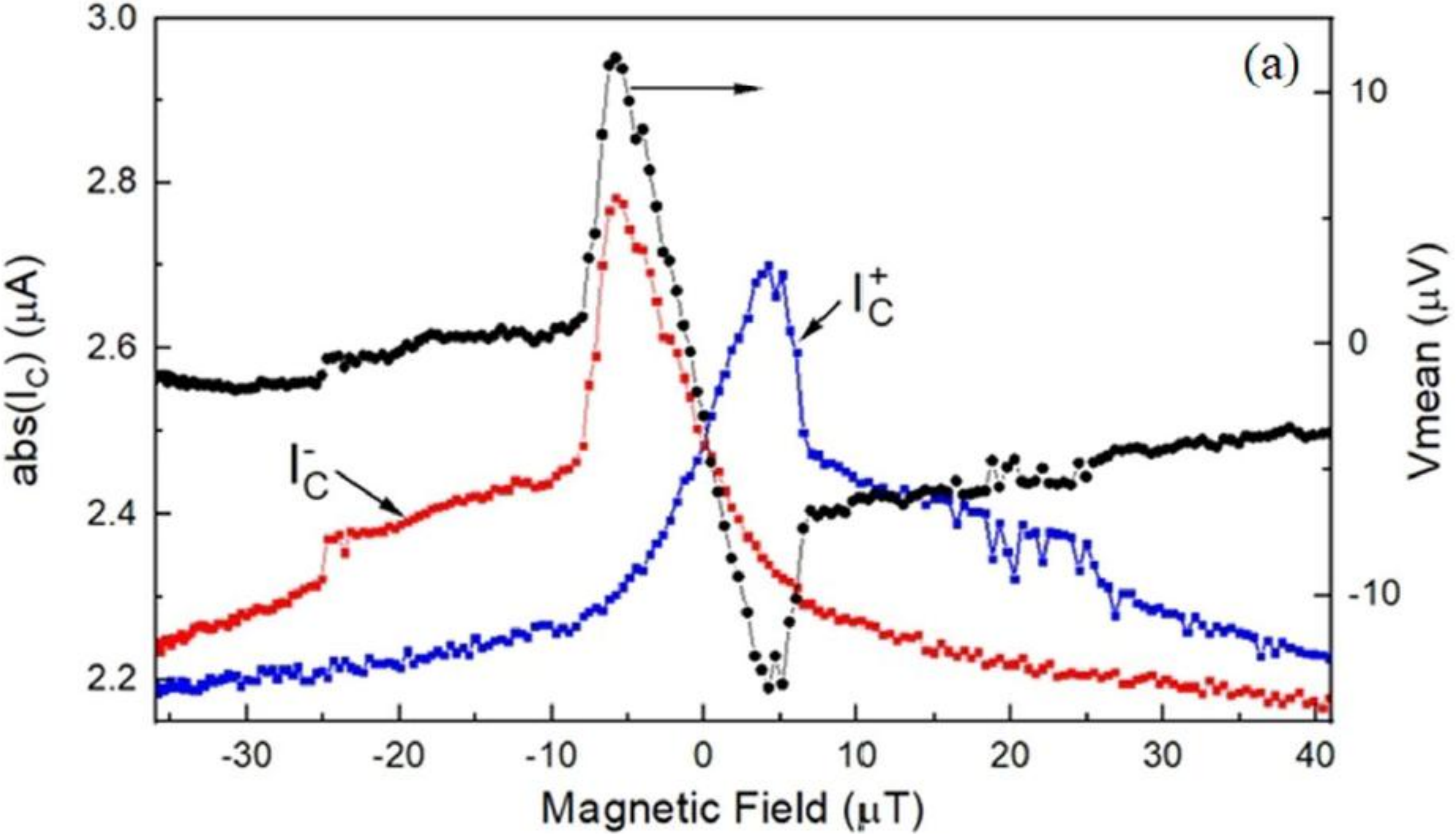
James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Diode effect with superconducting arrays



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin,
James N. Eckstein & Alexey Bezryadin
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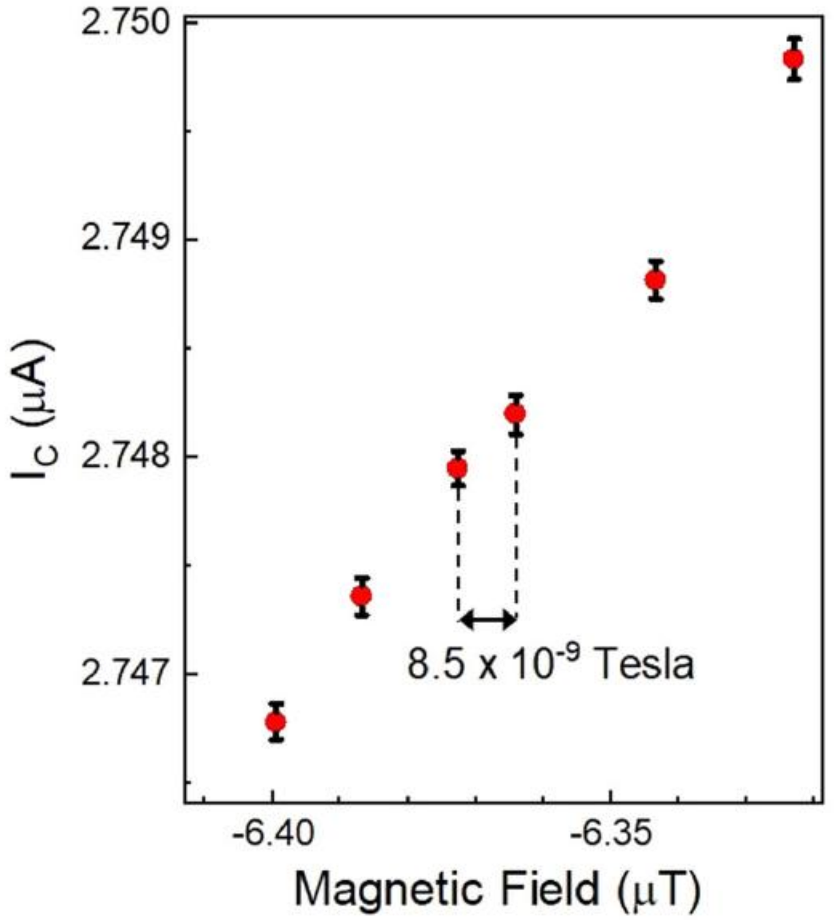
COMMUNICATIONS PHYSICS | (2023) 6:177 | https://doi.org/10.1038/s42005-023-01288-9 |www.nature.com/commsphys

Absolute magnetic field sensor based on superconducting array



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